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Introduction to Photovoltaics

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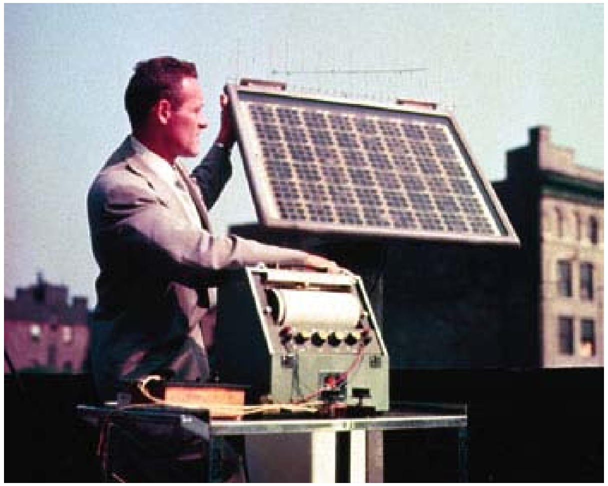
acknowledgement

Dionisis Berdebes, Jim Moore, and Xufeng Wang played key roles in putting together this tutorial. Their assistance is much appreciated.

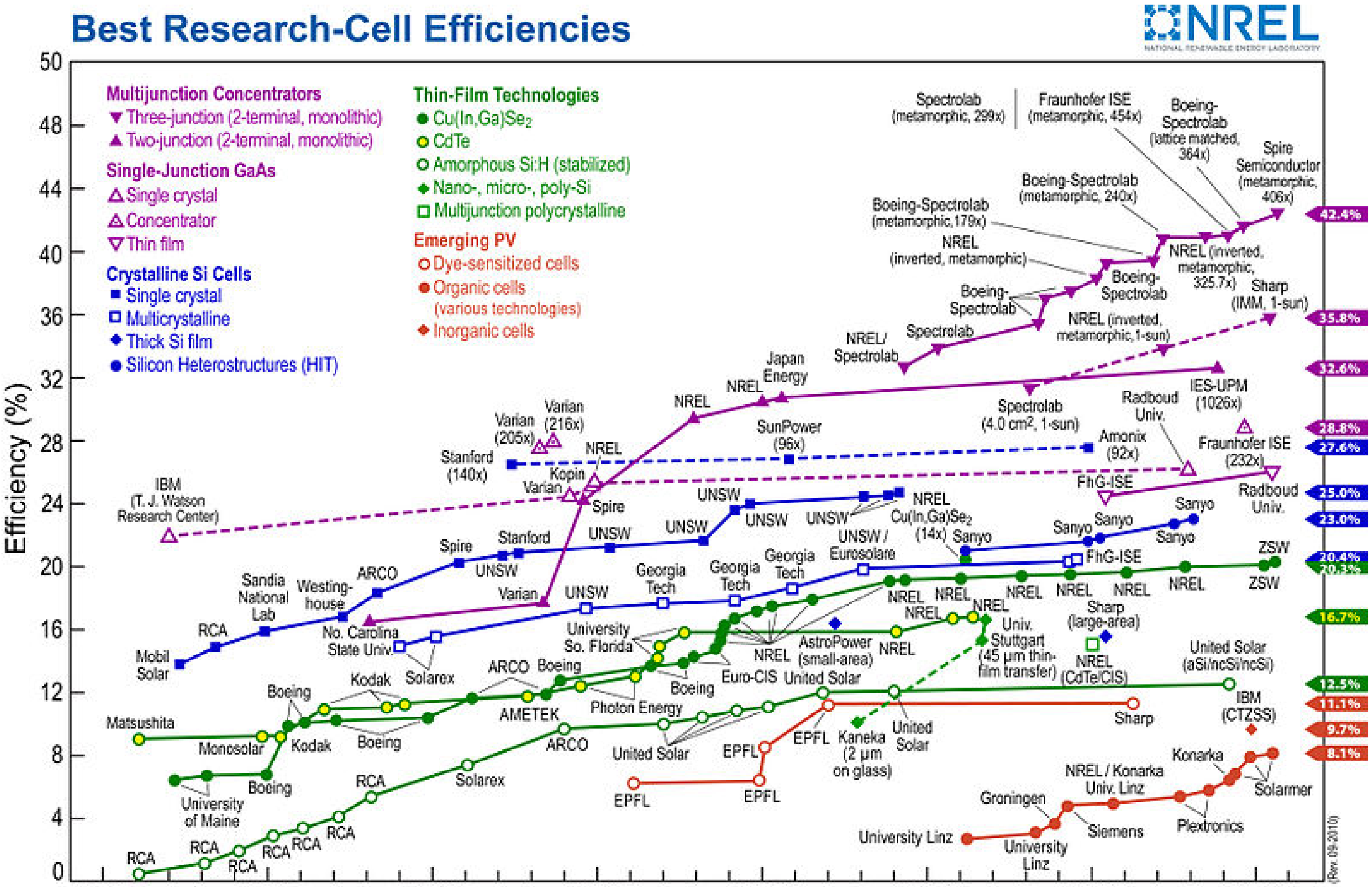
modern solar cell



Chapin, Pearson, and Fuller, Bell Labs. 1954

http://www.bell-labs.com/org/physicalsciences/timeline/span10.html#

solar cell progress



http://en.wikipedia.org/wiki/File:PVeff(rev100921).jpg

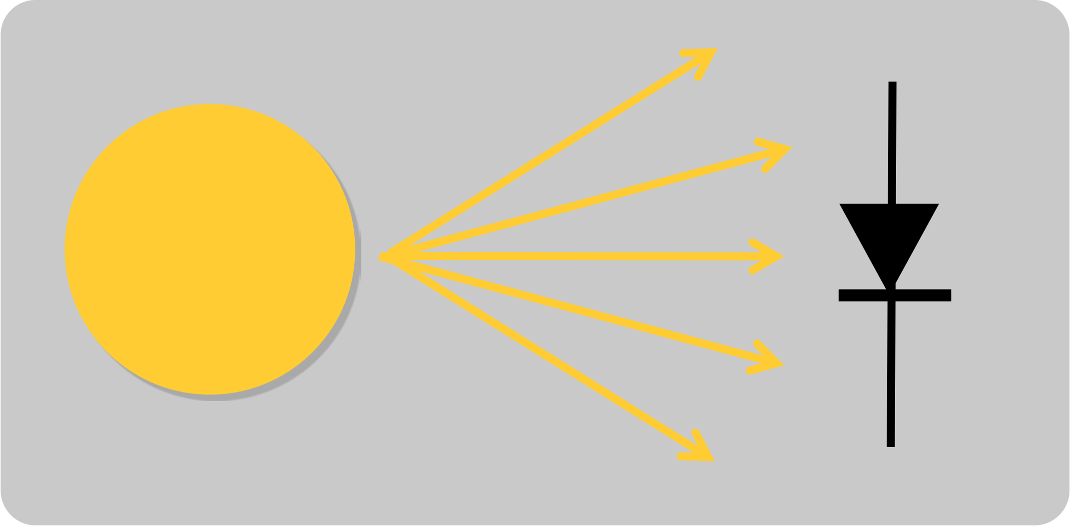
solar cell industry

SunPower / Applied Materials

Mark Pinto, “Renewable Solar Energy: Has the Sun Finally Risen on Photovoltaics?”
https://nanohub.org/resources/6332

solar cell

A solar cell is a junction (usually a PN junction) with sunlight shining on it.



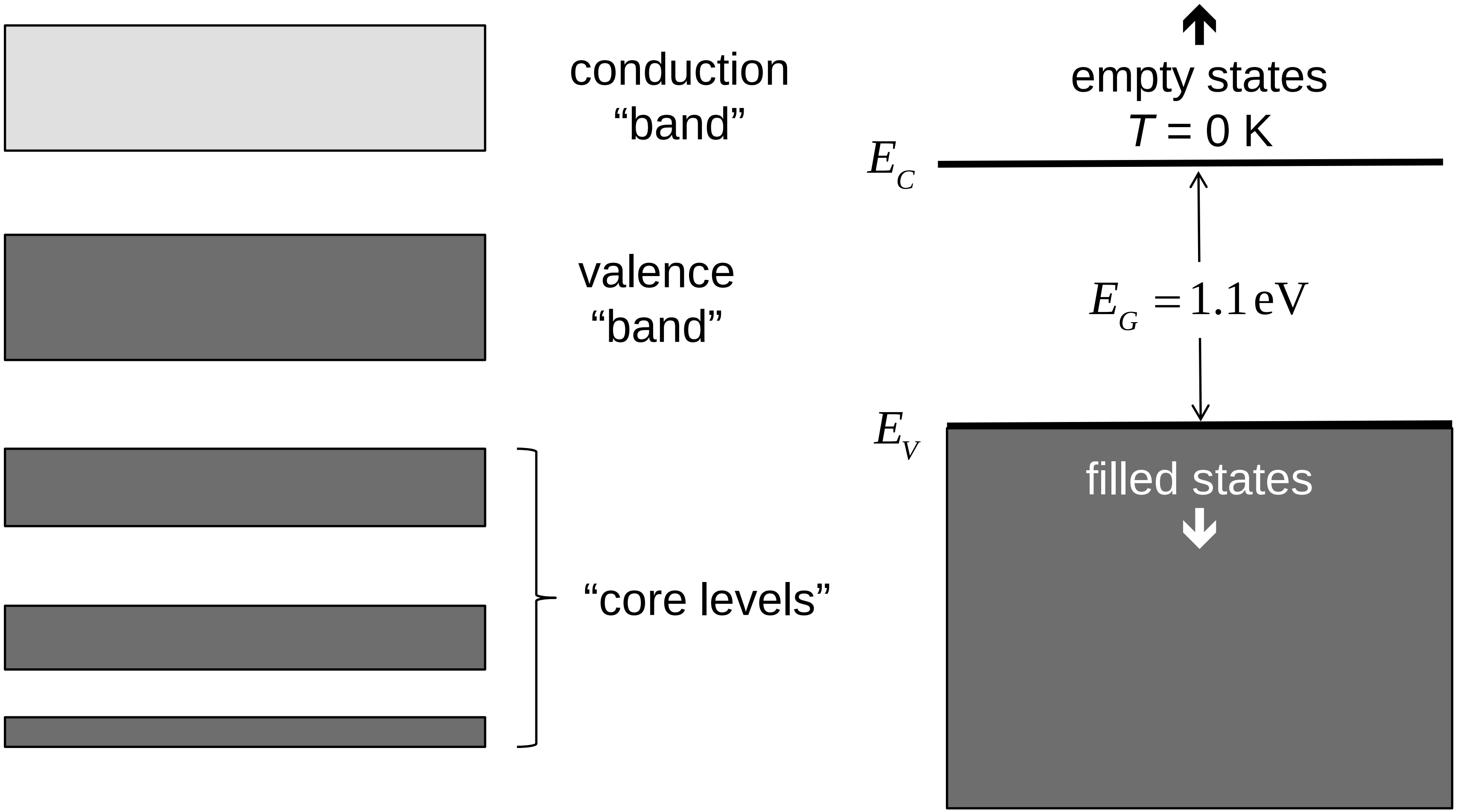
To understand how a solar cell works, we need to understand:

1) how a PN junction works (w/o the light)
2) how light is absorbed in a semiconductor (without a PN junction)
3) what happens when we put the two together.

outline

1) Introduction
2) **PN junction fundamentals (dark)**
3) Model solar cell: dark IV
4) Optical absorption / light-generated current
5) Model solar cell: illuminated
6) Discussion
7) Summary

silicon energy bands



conduction
“band”

valence
“band”

“core levels”

empty states
$T = 0$ K

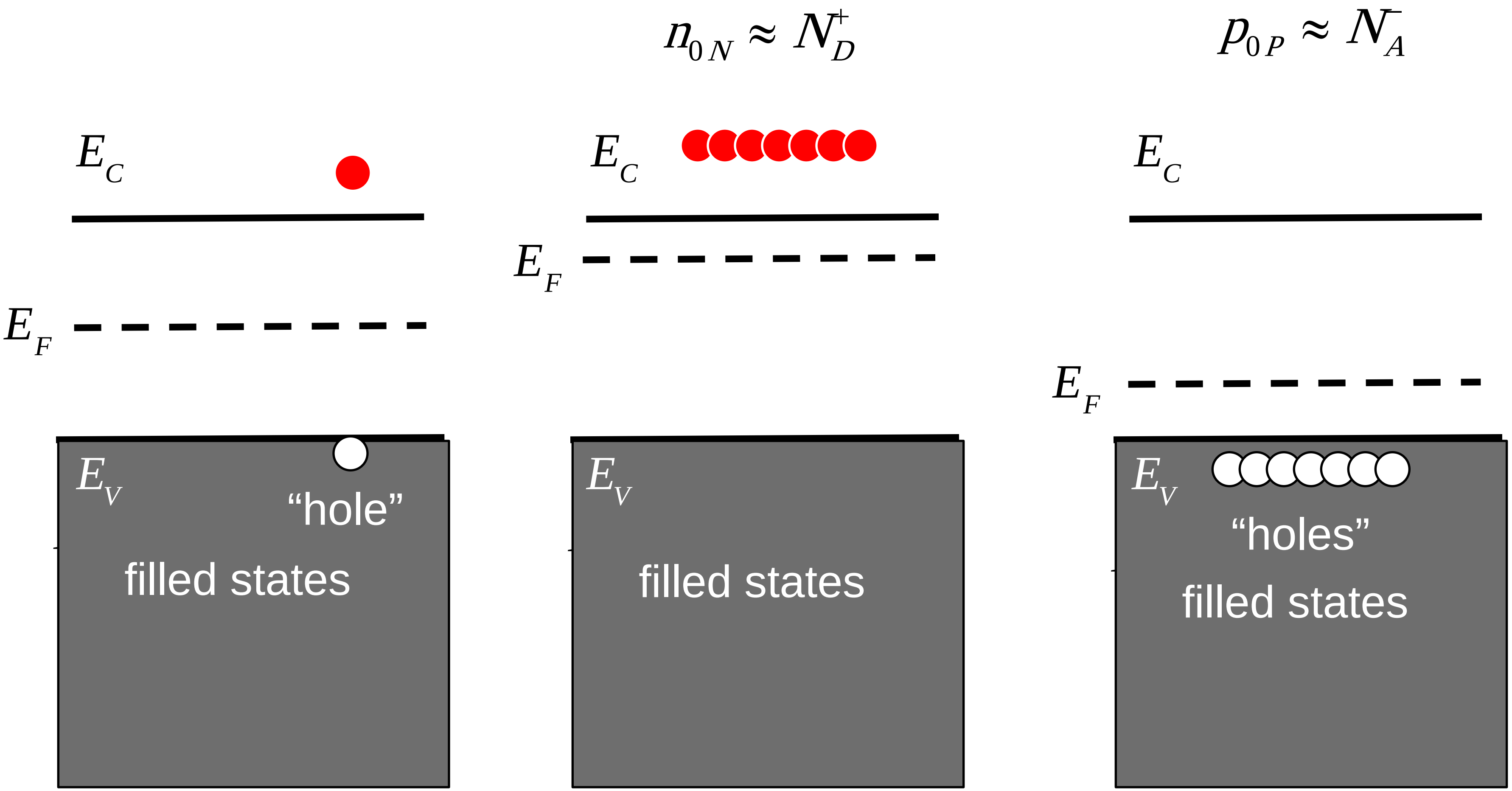
E_C

$E_G = 1.1\,\mathrm{eV}$

E_V

filled states

intrinsic n-type p-type



$n_{0N} \approx N_D^+$

$p_{0P} \approx N_A^-$

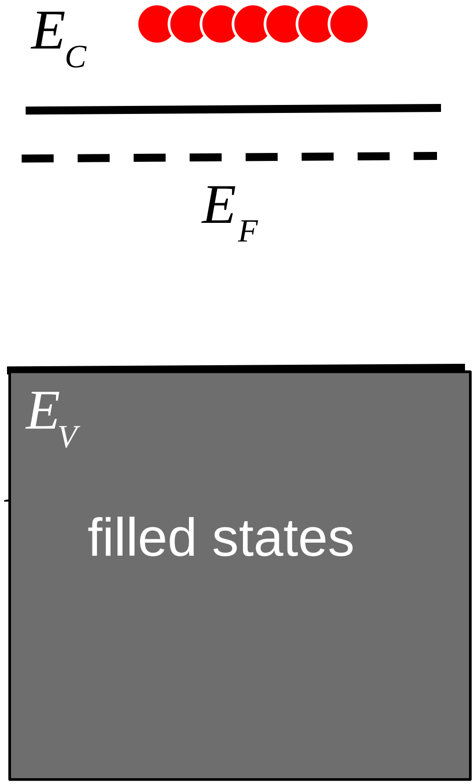
intrinsic: E_C E_F E_V “hole” filled states

n-type: E_C E_F E_V filled states

p-type: E_C E_F E_V “holes” filled states

Fermi level

$$n_{0N} \approx N_D^+$$



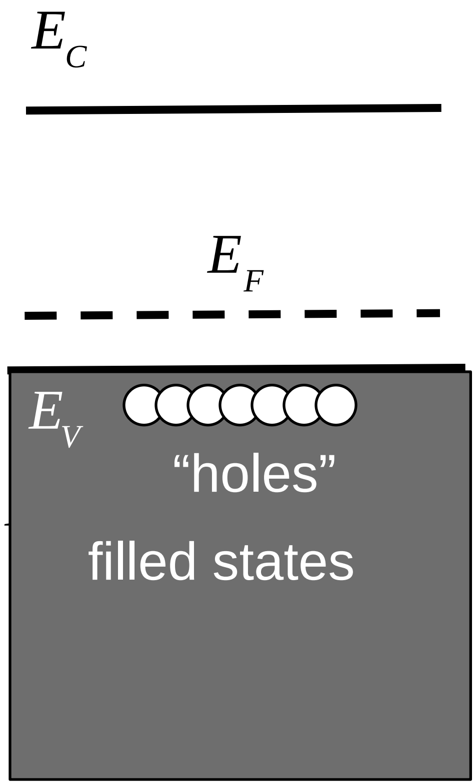
$$f_0(E) = \frac{1}{1 + e^{(E - E_F)/k_B T}}$$

$$E = E_F \rightarrow f_0(E) = \frac{1}{2}$$

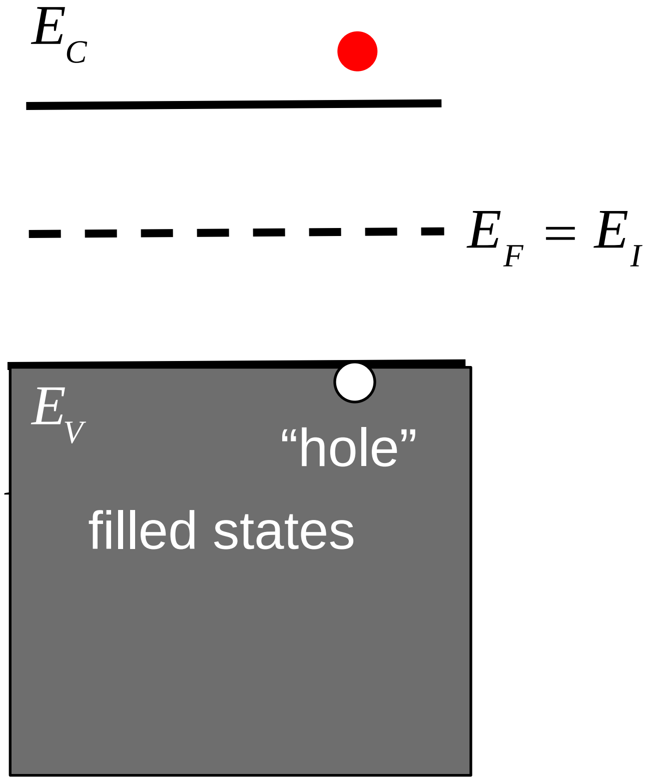
$$E << E_F \rightarrow f_0(E) = 1$$

$$E >> E_F \rightarrow f_0(E) = 0$$

$$p_{0P} \approx N_A^-$$



intrinsic semiconductor



$$n_0 = n_i$$

$$p_0 = n_i$$

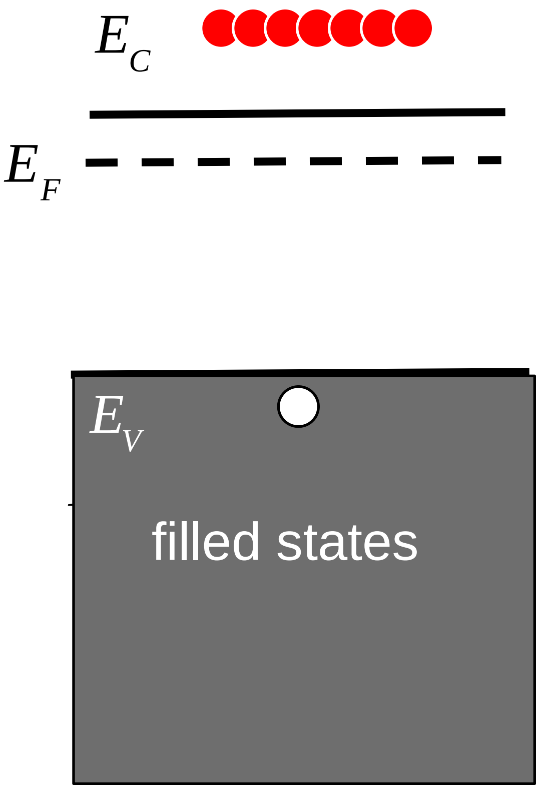
$$n_0 p_0 = n_i^2$$

$$n_i \propto e^{-E_G/k_B T_L}$$

$$n_i(300\,K) \approx 10^{10} \text{ cm}^{-3}$$

n-type semiconductor

$$n_{0N} \approx N_D^+$$



$$n_0 \approx N_D^+$$

$$n_0 p_0 = n_i^2$$

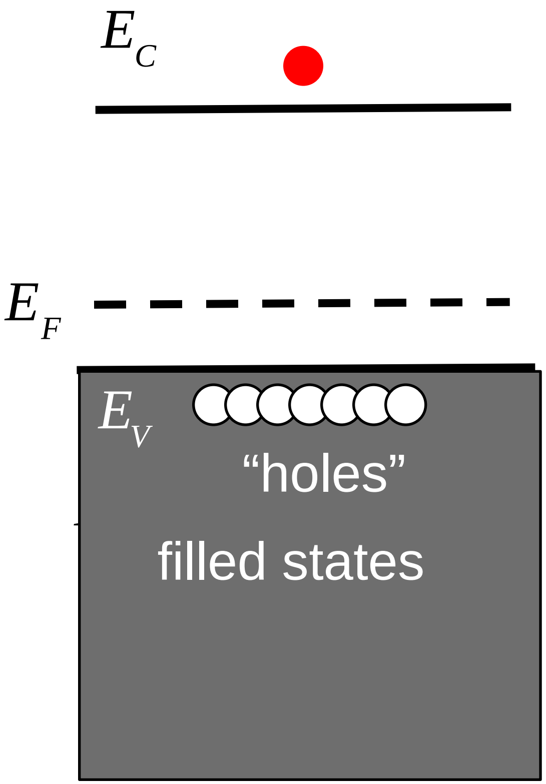
$$p_0 = n_i^2 / N_D^+$$

$$n_0 = N_D \approx 10^{17} \text{ cm}^{-3}$$

$$n_i(300\,K) \approx 10^{10} \text{ cm}^{-3}$$

$$p_0 \approx 10^{3} \text{ cm}^{-3}$$

p-type semiconductor



$$p_0 \approx N_A^-$$

$$n_0 p_0 = n_i^2$$

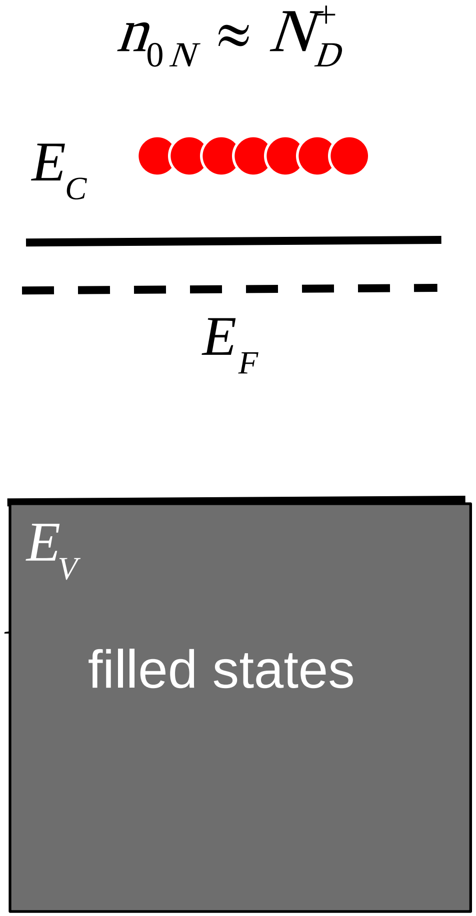
$$n_0 = n_i^2 / N_A^-$$

$$p_0 = N_A \approx 10^{17} \text{ cm}^{-3}$$

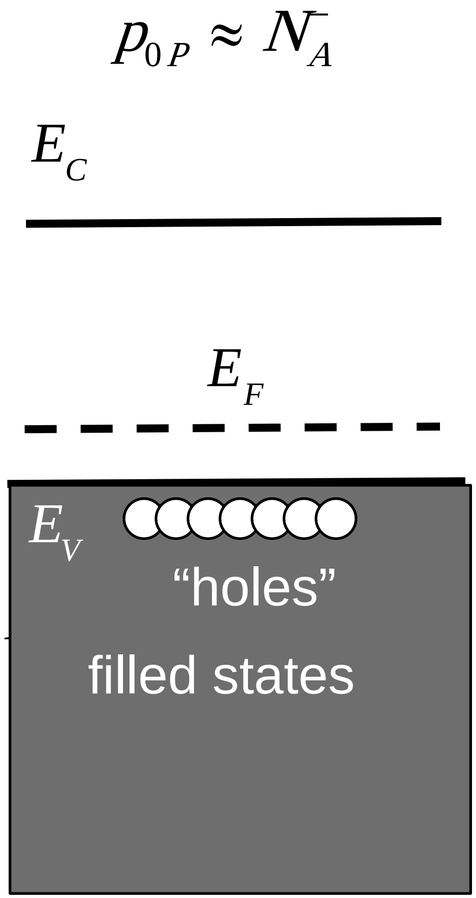
$$n_i(300\,K) \approx 10^{10} \text{ cm}^{-3}$$

$$n_0 \approx 10^{3} \text{ cm}^{-3}$$

PN junction



What happens if we bring the p and n regions together?

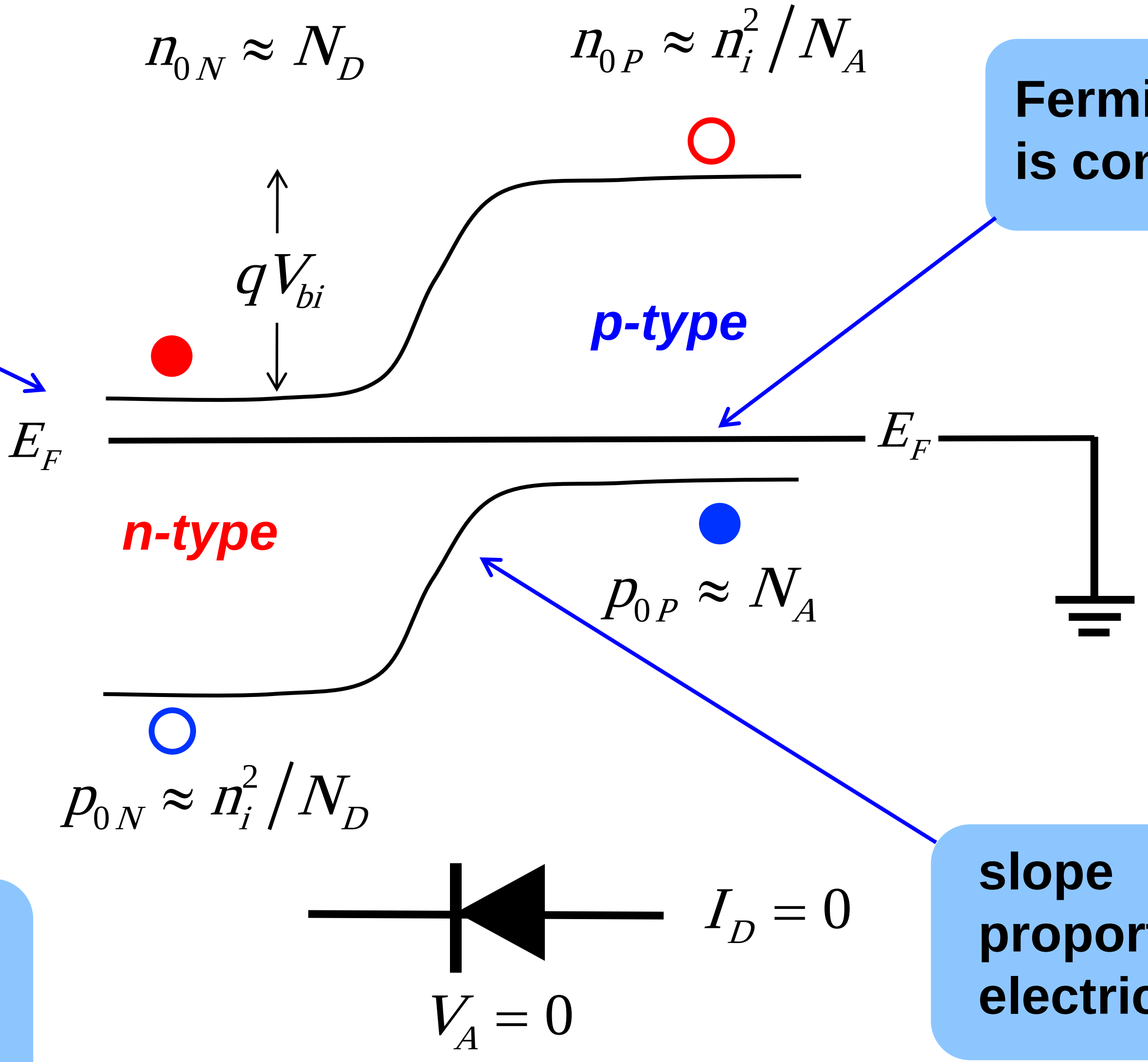


PN junction in equilibrium

E_C lower

$E_C = -qV$

$V = V_{bi}$



$n_{0N} \approx N_D$

$n_{0P} \approx n_i^2 / N_A$

$p_{0P} \approx N_A$

$p_{0N} \approx n_i^2 / N_D$

$V = 0$

$I_D = 0$

$V_A = 0$

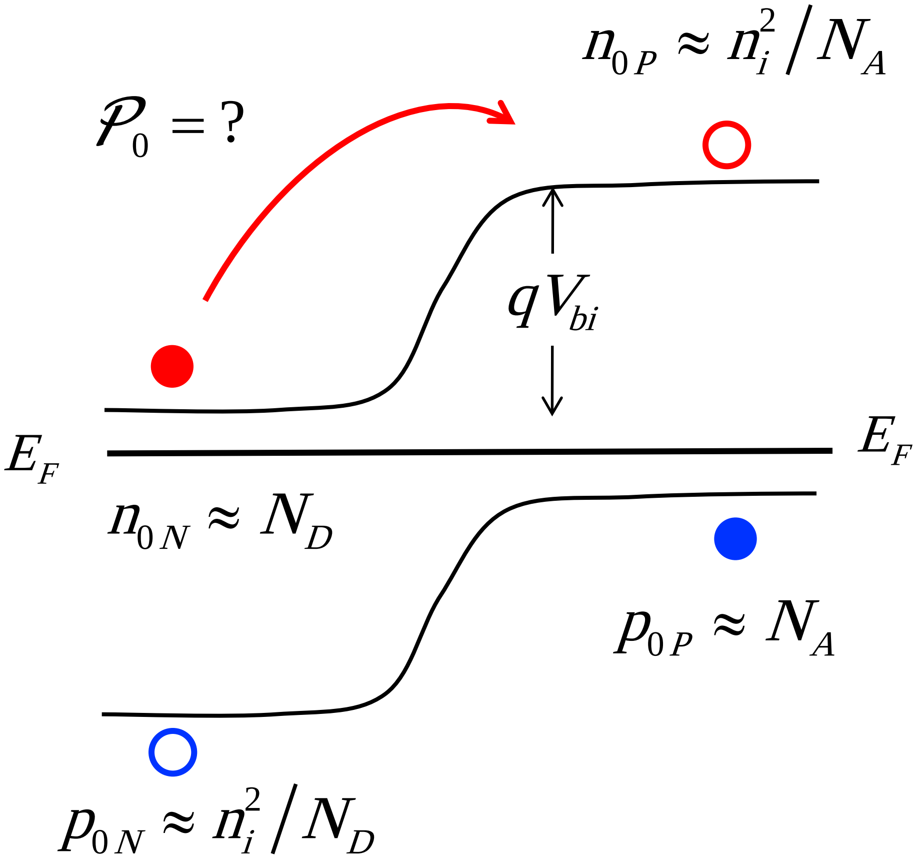
Fermi level is constant!

slope proportional to electric field

V_{bi} ~ E_G

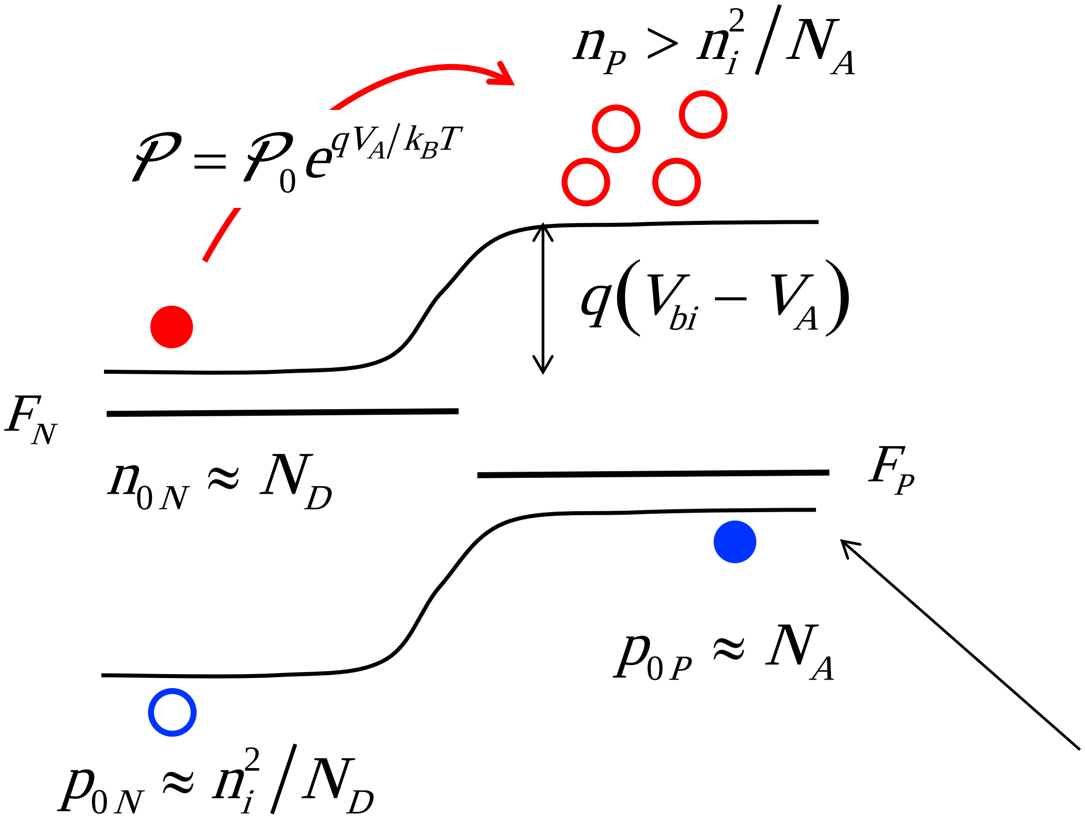
$$qV_{bi} = \frac{k_B T}{q} \ln \frac{N_A N_D}{n_i^2}$$

PN junction in equilibrium



$$\mathcal{P}_0 = e^{-\Delta E / k_B T}$$

PN junction under forward bias



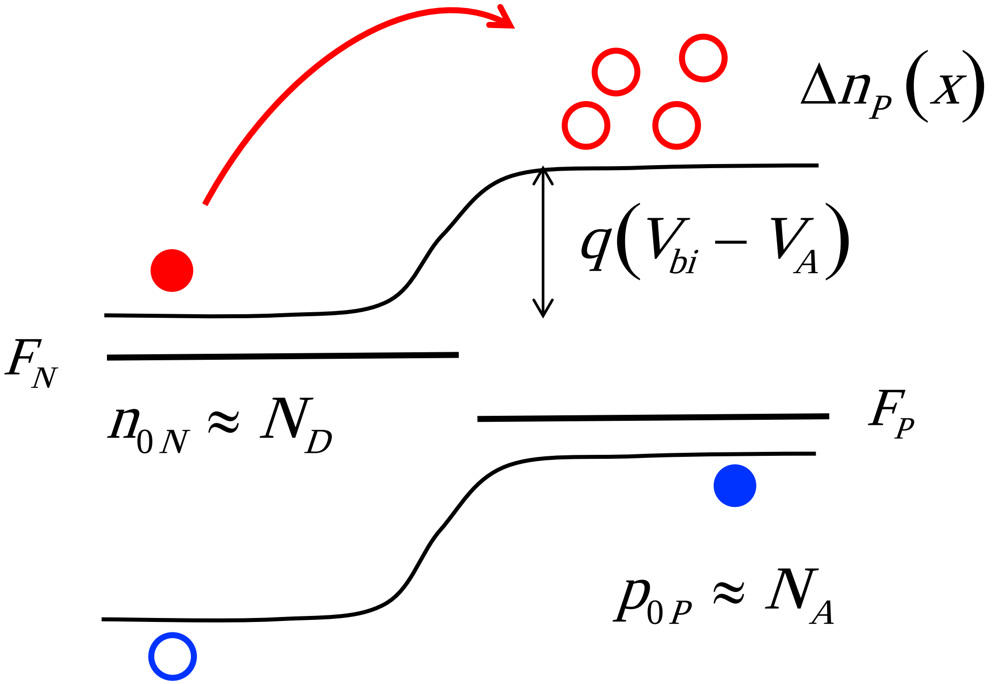
$$\mathcal{P} = e^{-q\left(V_{bi} - V_A\right)/k_B T}$$

“excess carriers”

A positive voltage on p-side pulls the bands down.

Energy barrier is lowered.

PN junction under forward bias



Excess electron concentration on the p-side.

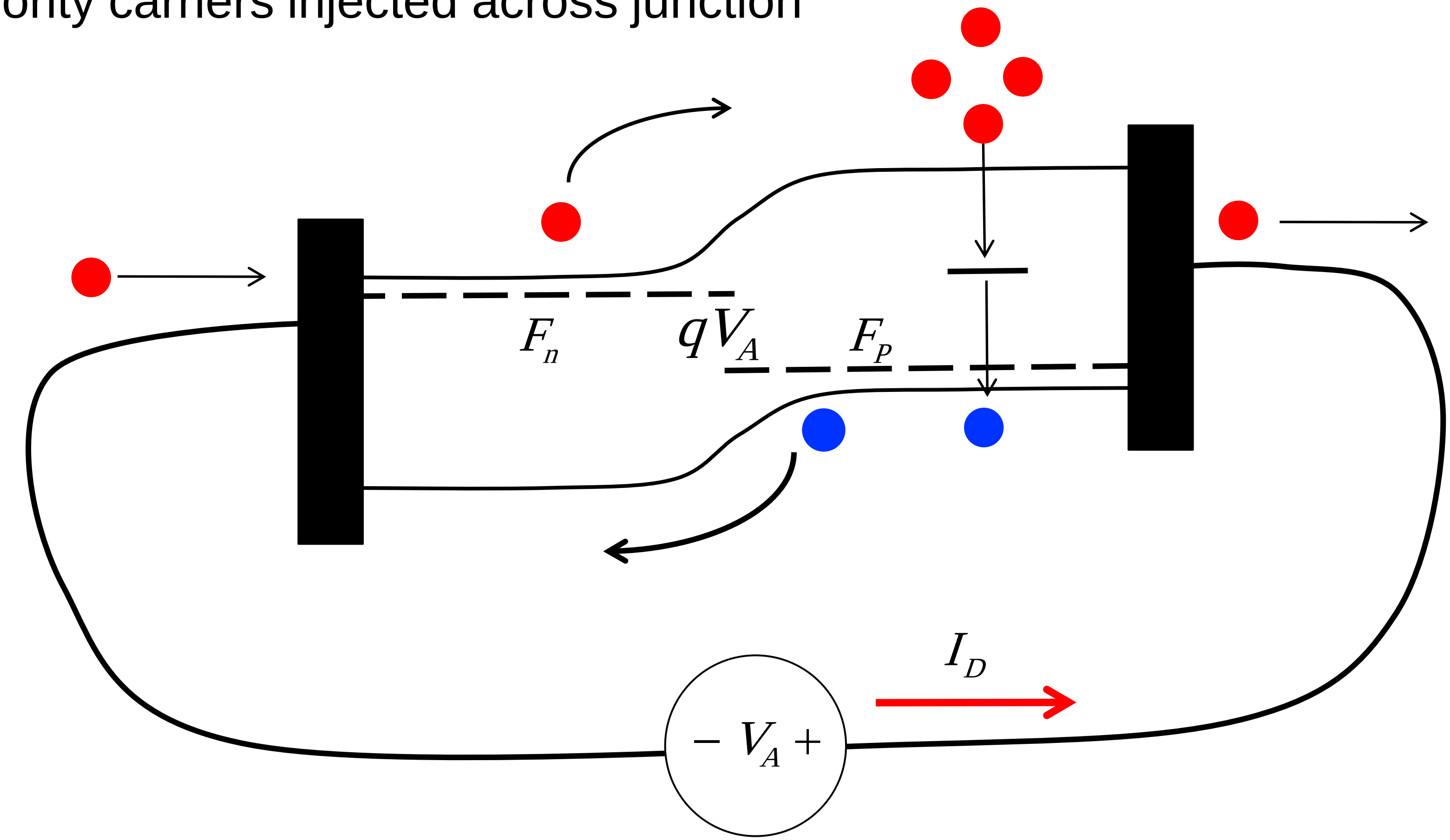
$$Q_n = q\int_0^{W_P} \Delta n_P(x)\,dx$$

$$I_D = \frac{Q_n}{t_n} + \frac{Q_p}{t_p}$$

The time, t_n is the average time for an electron on the p-side to "recombine" or to diffuse to the contact and recombine. **Let's see why recombination leads to current.**

recombination leads to current

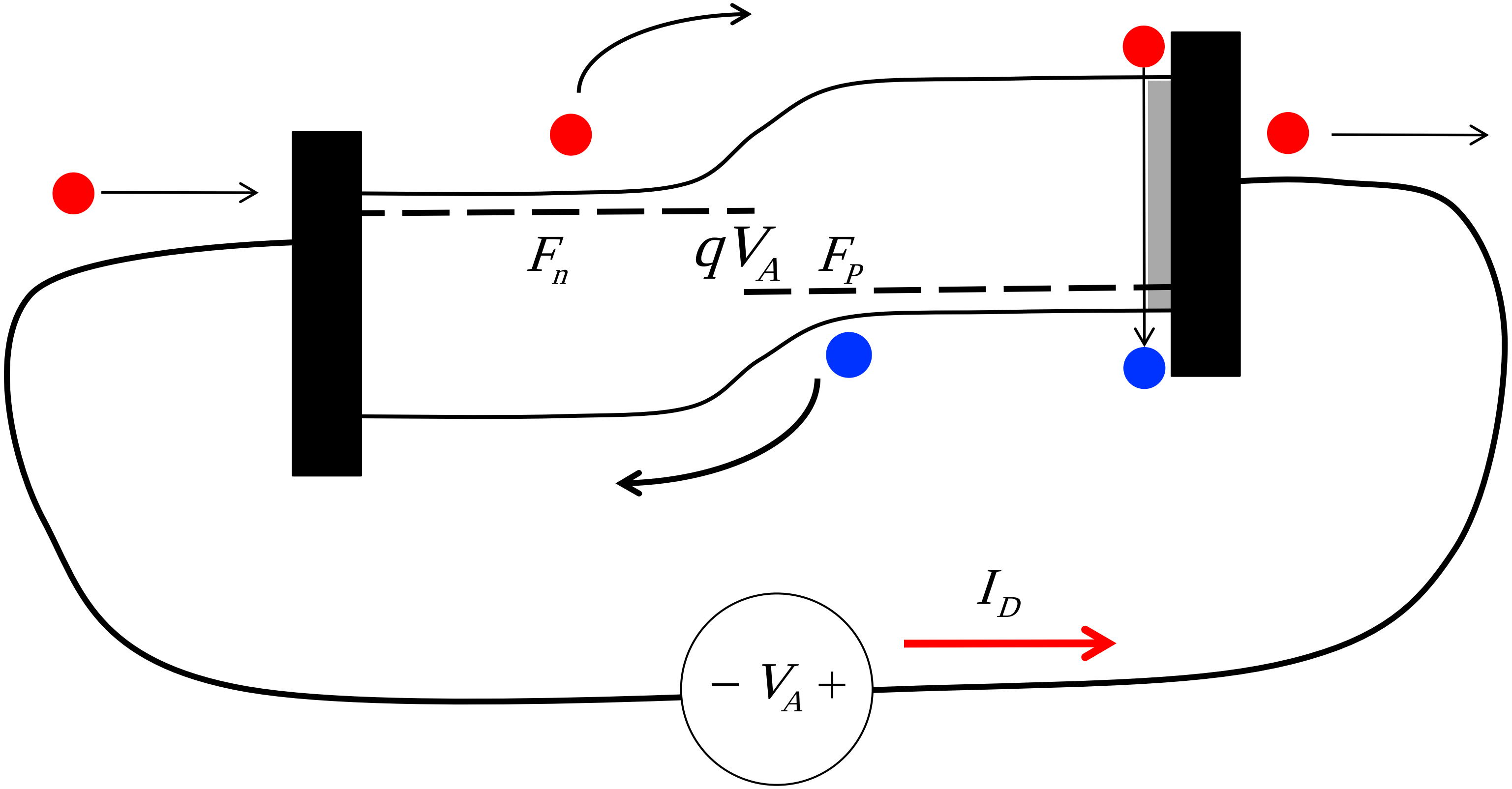
minority carriers injected across junction



Every time a minority electron recombines on the p-side, one electron flows in the external current.

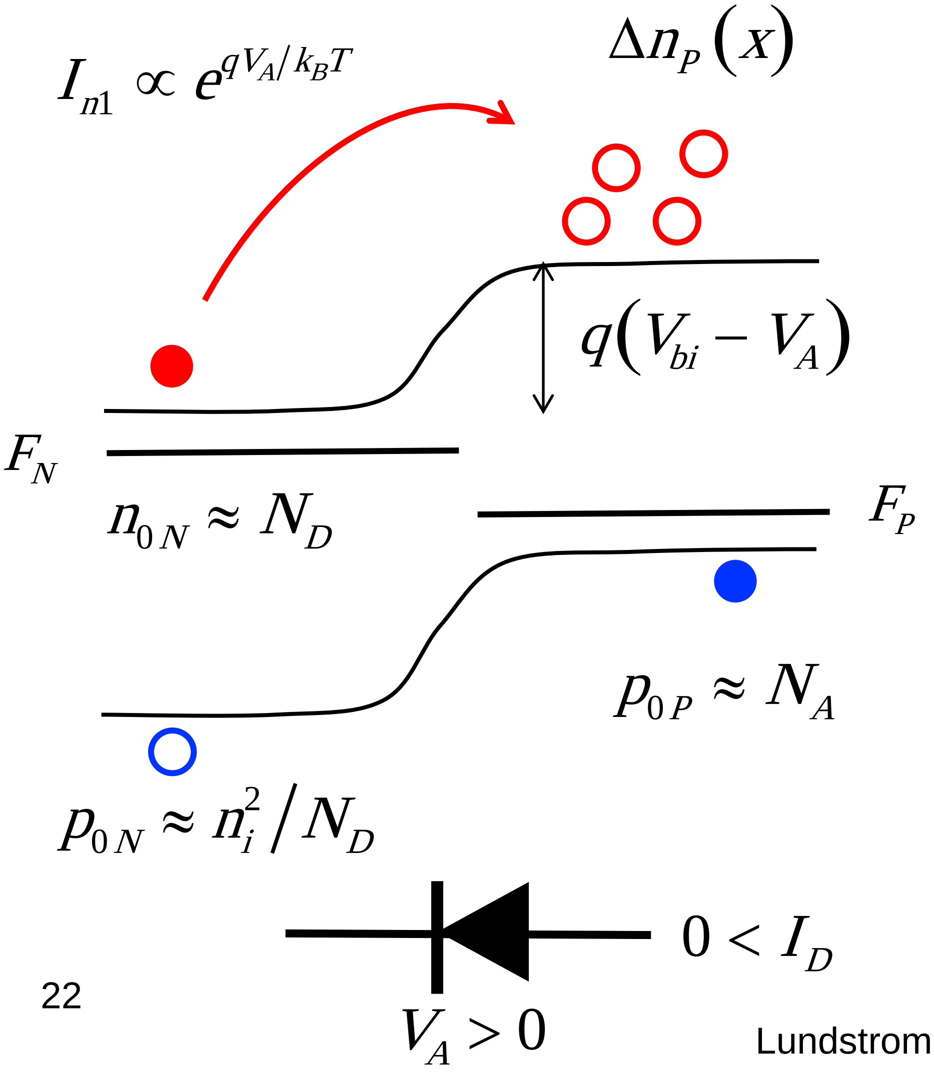
recombination at a contact

minority carriers injected across junction



Every time a minority electron recombines on the p-side, one electron flows in the external current.

forward bias summary



1) Injected current produces a population of “excess” electrons in the P-type region...

2) Excess electrons in the P-type region recombine…

3) Every time an electron and hole recombine, an electron flows in the external circuit.

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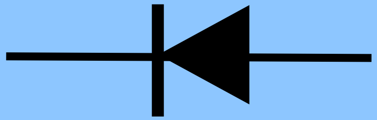
ideal diode equation

$$I_D = \frac{Q_n}{t_n} + \frac{Q_p}{t_p}$$



$$Q_n \propto \frac{n_i^2}{N_A}\left(e^{qV_A/k_BT} - 1\right)$$

$$I_D = I_0\left(e^{qV_A/k_BT} - 1\right)$$

"ideal diode equation"

$$I_D = I_0\left(e^{qV_A/nk_BT} - 1\right)$$

$$n = 1$$

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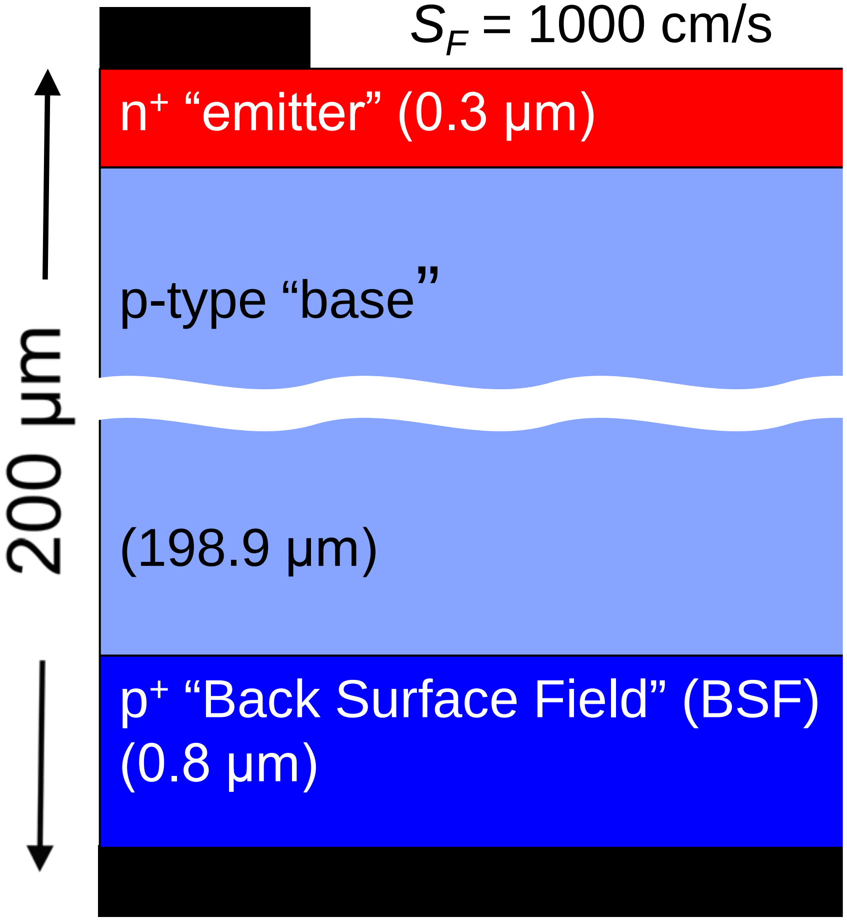
generic crystalline Si solar cell



S_F = 1000 cm/s

n^+ “emitter” (0.3 µm)

p-type “base”

(198.9 µm)

p^+ “Back Surface Field” (BSF) (0.8 µm)

200 µm

key device parameters

base doping: $N_A = 10^{16}$ /cm^3

emitter doping $N_D = 6 \times 10^{19}$ /cm^3

minority carrier lifetime τ_n = 34 µs (base)

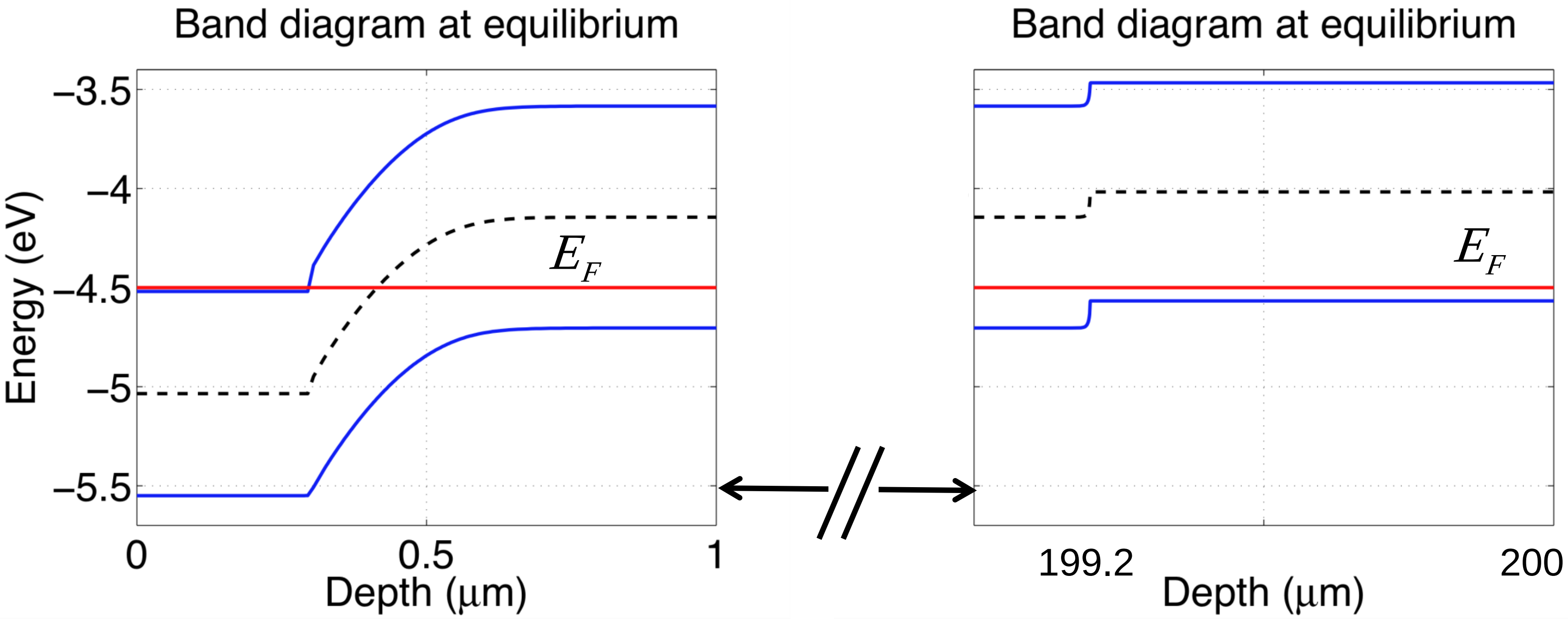
base thickness W = 198.9 µm

front junction depth x_{jf} = 0.3 µm

back junction depth x_{jb} = 0.8 µm

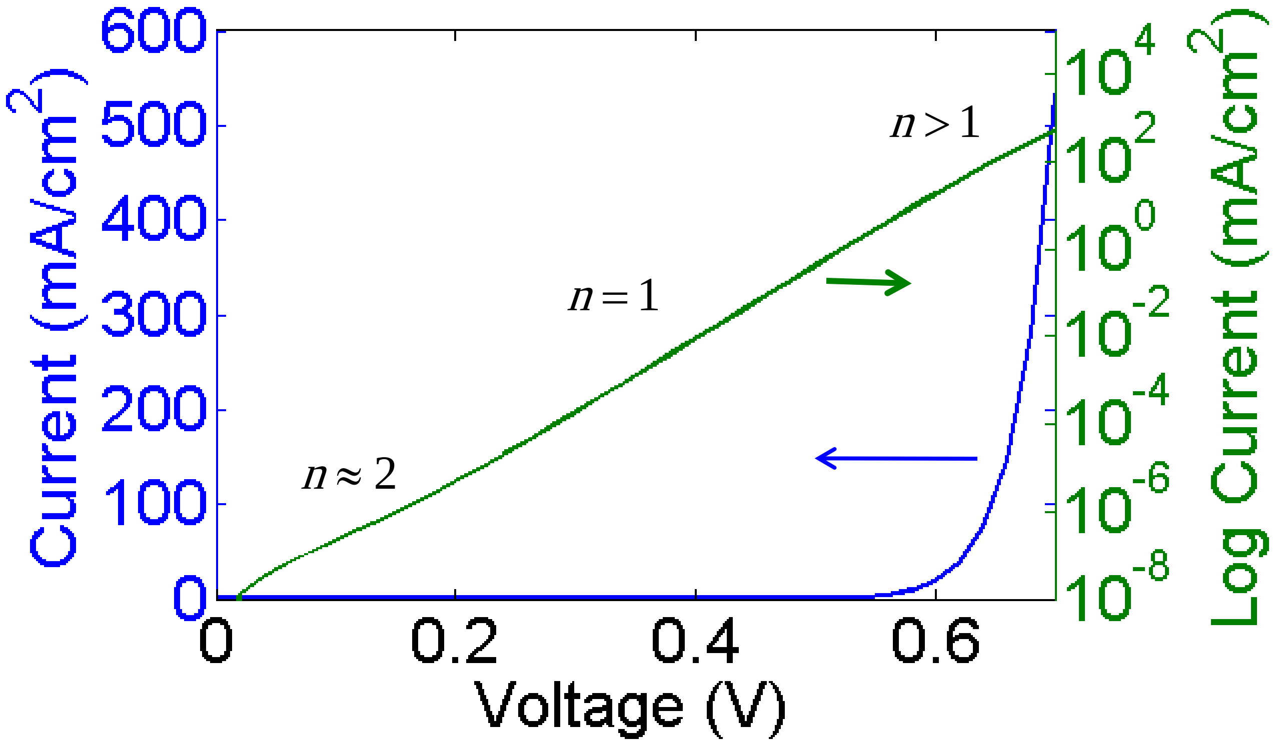
R_s = 1 Ohm-cm^2

equilibrium e-band diagram



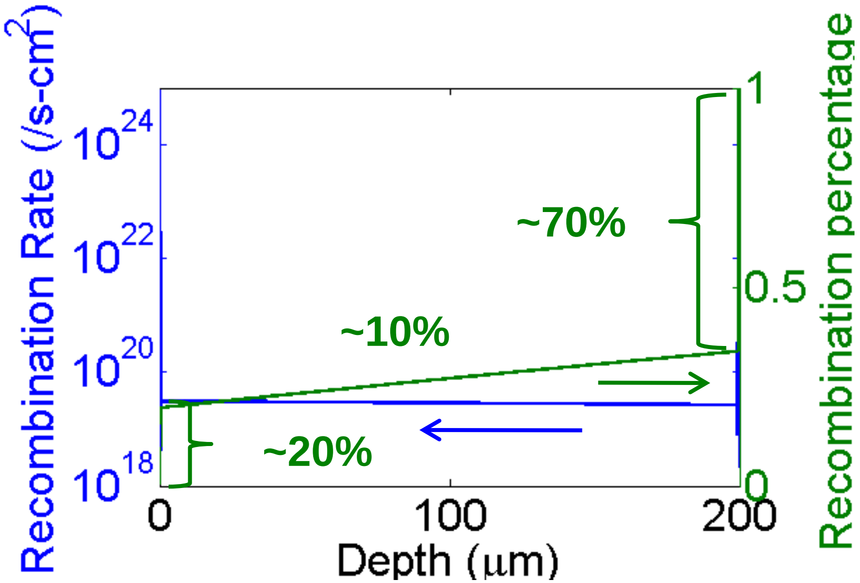
dark I-V

$$J_D = J_0\left(e^{qV_A/nk_BT} - 1\right)$$

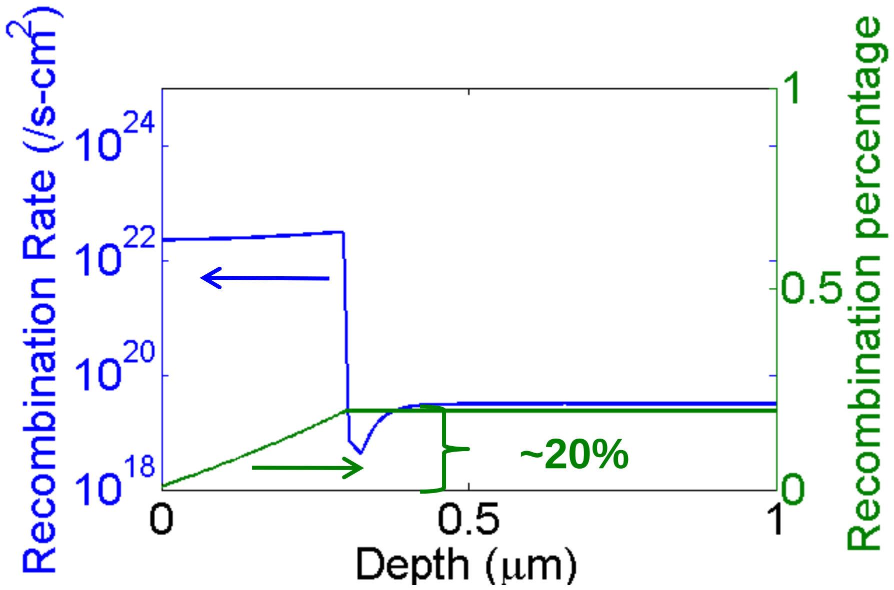


recombination: *V* = 0.7 V

entire device



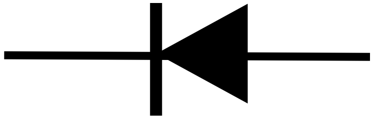
front surface region



$R(x)$

$$\frac{\int_0^x R(x')dx'}{\int_0^L R(x')dx'}$$

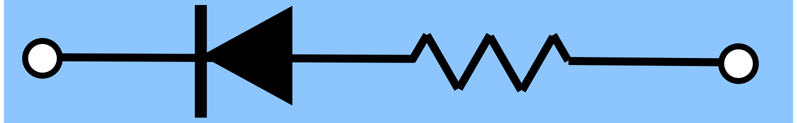
series resistance


$0 < I_D$

$- \; V_A \; +$

$$I_D = I_0\left(e^{qV_A/k_BT} - 1\right)$$

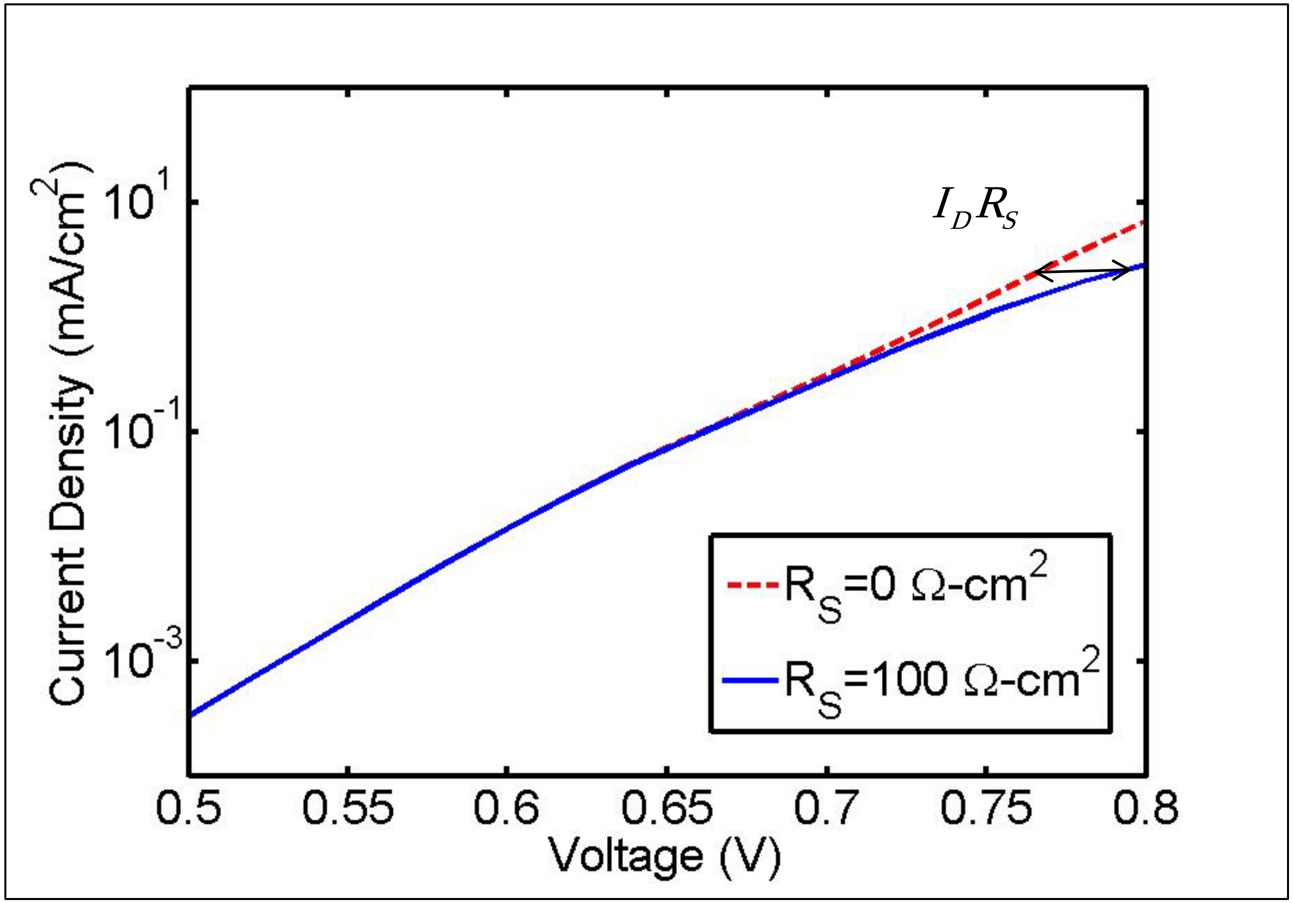
$- \; V_A \; +$ $\qquad R_S$



$$\longleftarrow \quad V_D = V_A + I_D R_S \quad \text{——}$$

$$I_D = I_0\left(e^{q\left(V_D - I_D R_S\right)/k_BT} - 1\right)$$

dark I-V and series resistance



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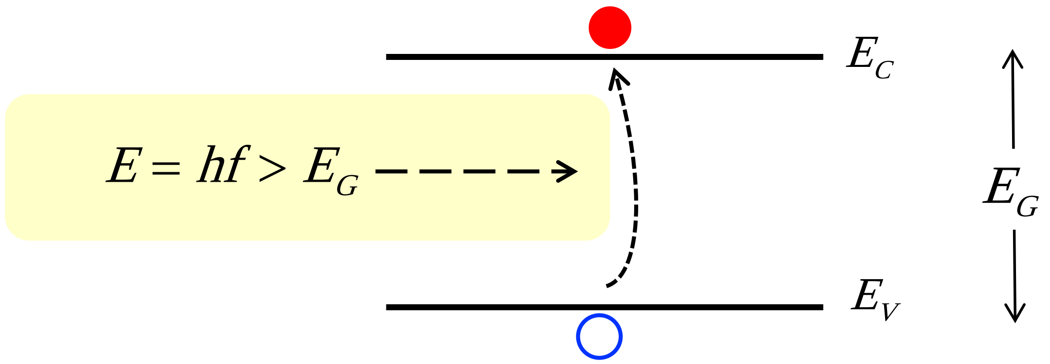
absorption of light

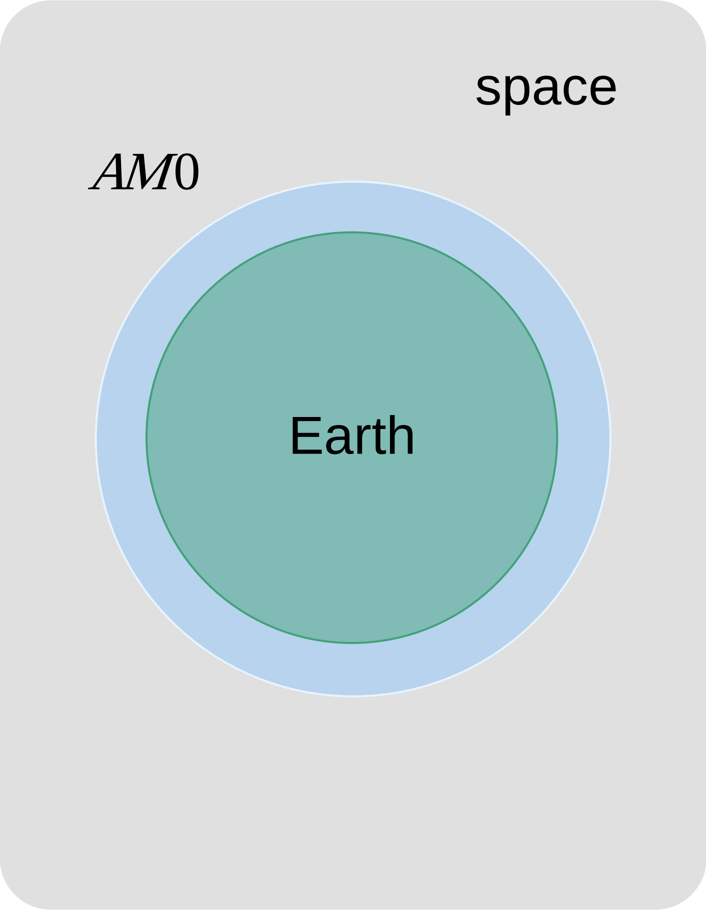
$$\lambda < \frac{hc}{E_G}$$



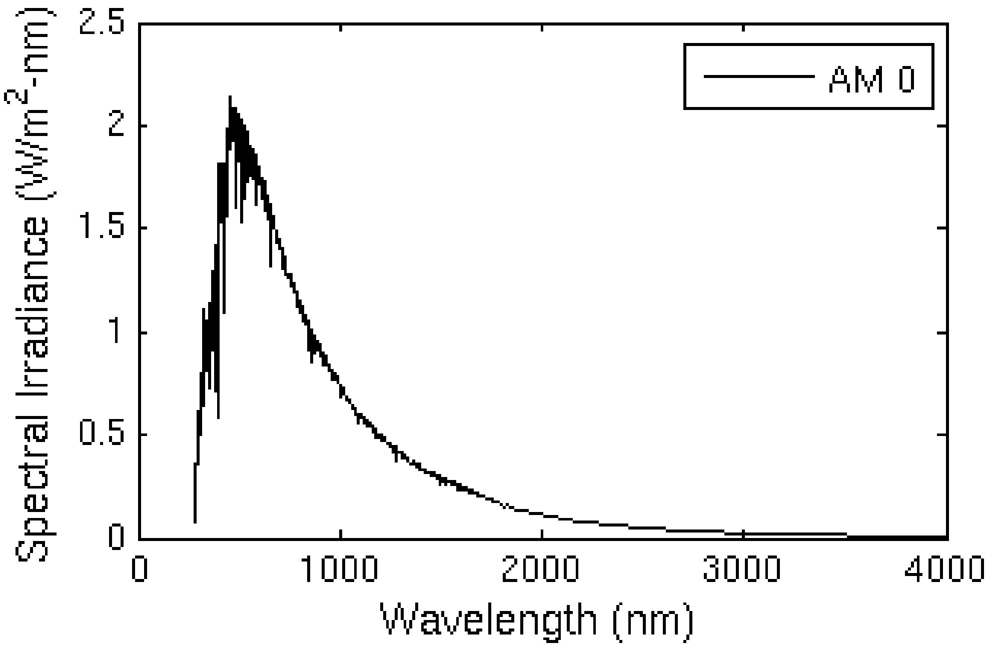
$$E = hf > E_G$$

$$f\lambda = c \qquad f = \frac{c}{\lambda} \qquad E = hf = \frac{hc}{\lambda}$$

solar spectrum (outer space)

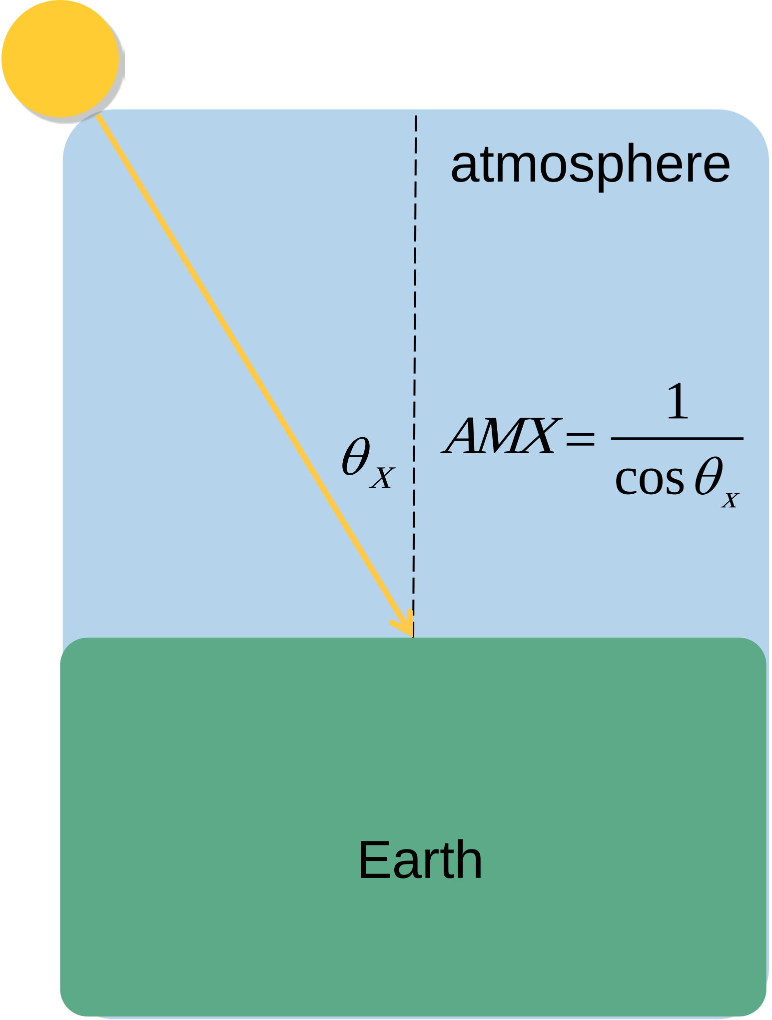


AM0 = “air mass 0”



Integrated power = 136.6 mW/cm^2

solar spectrum (terrestrial)

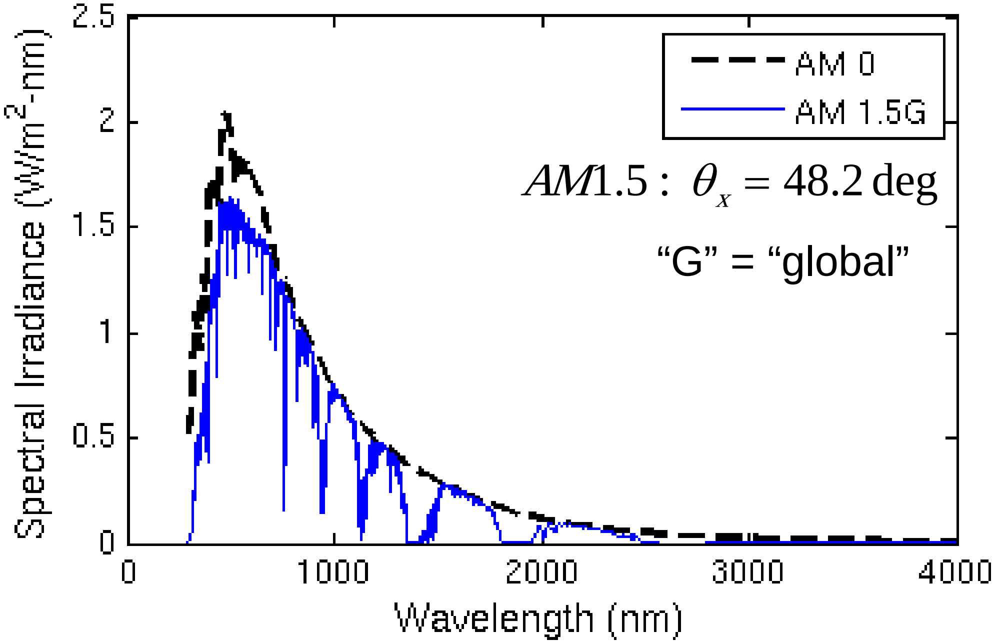


atmosphere

θ_X

$$AMX = \frac{1}{\cos\theta_X}$$

Earth



$AM1.5: \ \theta_x = 48.2\,\text{deg}$

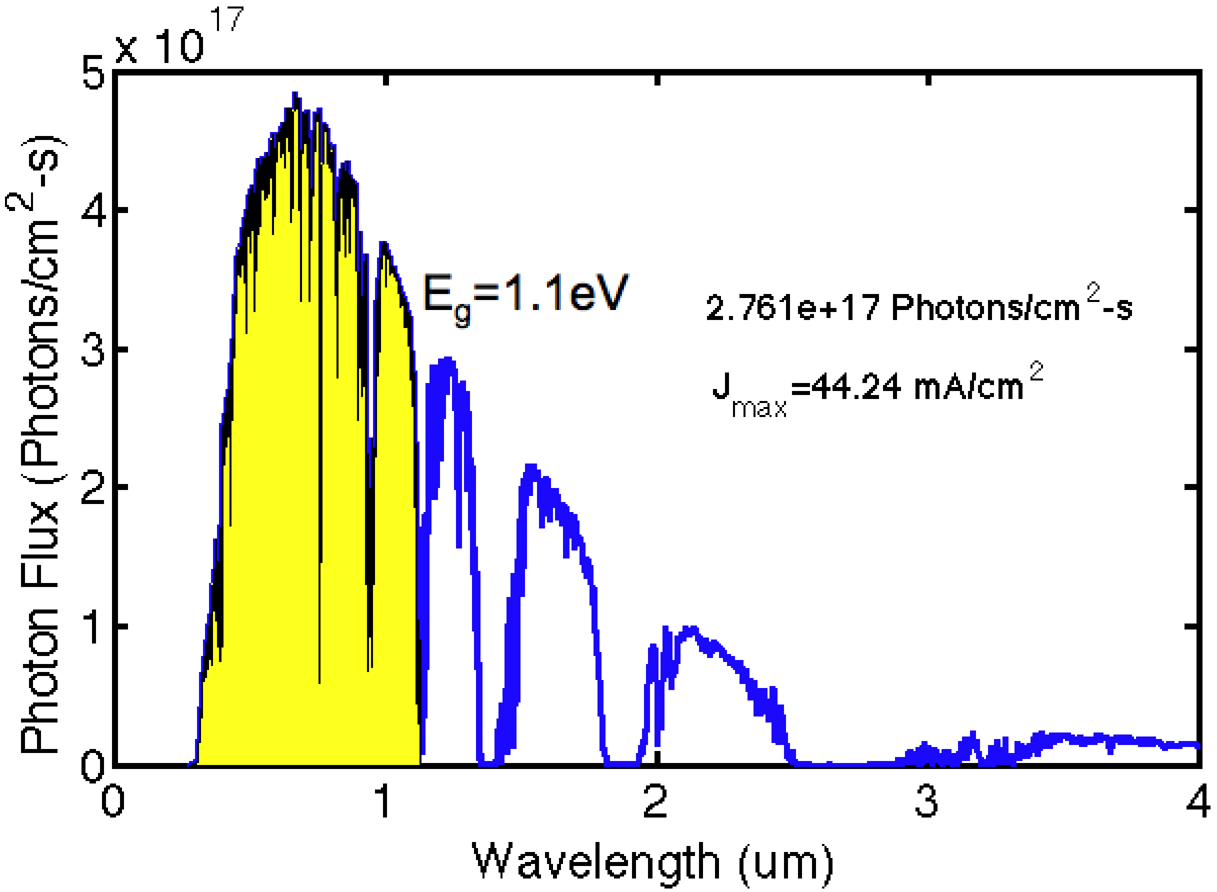
"G" = "global"

Integrated power = 100 mW/cm^2)

http://en.wikipedia.org/wiki/Air_mass_(solar_energy)

how many photons can be absorbed?

Example: Silicon E_g = 1.1eV. Only photons with a wavelength smaller than 1.1 μm will be absorbed.



solar spectrum (AM1.5G)

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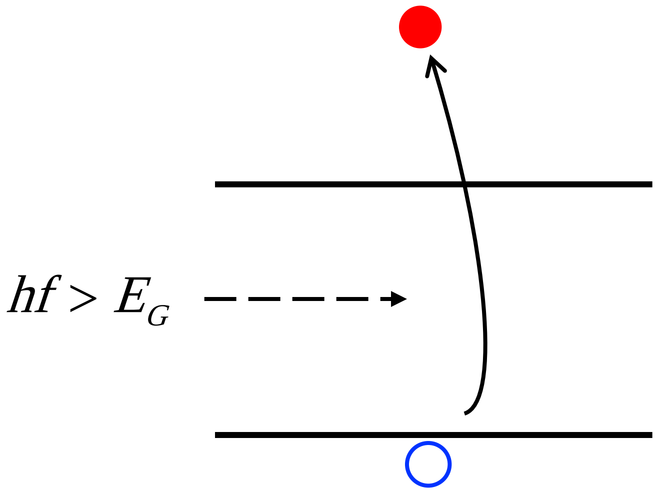
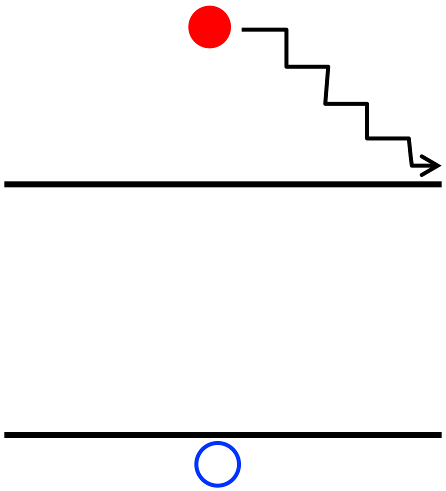
wasted energy for $E > E_G$

Energy is lost for photons with energy greater than the bandgap.



Electron is excited above the conduction band.



However, extra energy is lost due to thermalization as electron relaxes back to the band edge.

how many photons are absorbed in a finite thickness?

Incident flux: Φ_0

Flux at position, *x* : $\Phi(x) = \Phi_0 e^{-\alpha(\lambda)x}$

optical absorption coefficient:

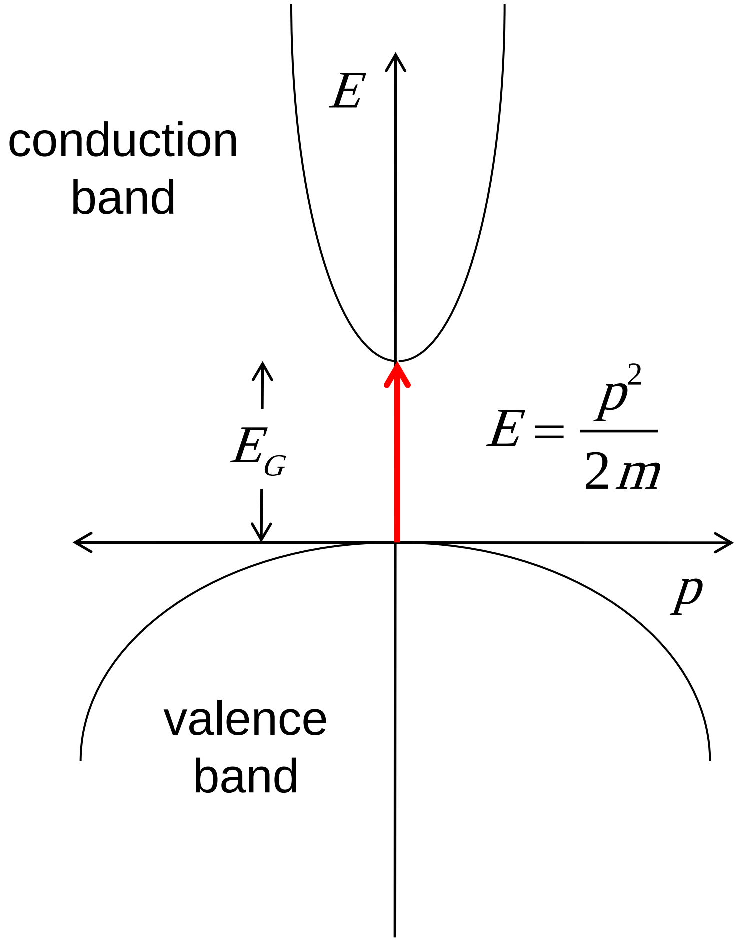
$$\alpha(\lambda) > 0 \quad \text{for} \quad E > E_G \left(\lambda < hc/E_G\right)$$

Generation rate at position, *x* :

$$G(x) = -\frac{d\Phi(x)}{dx} = \Phi_0 \alpha(\lambda) e^{-\alpha(\lambda)x}$$

$$G_{tot} = \int \left\{ \int_0^L G(x,\lambda)\, dx \right\} d\lambda$$

what determines alpha?

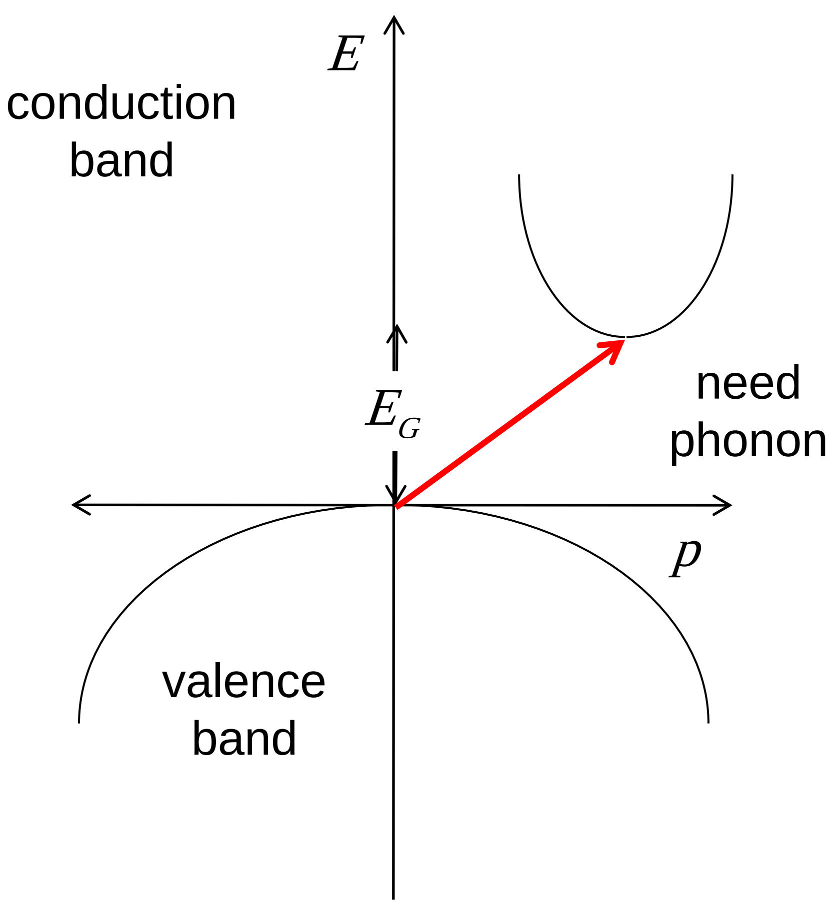


conduction band

$$E = \frac{p^2}{2m}$$

valence band

Direct gap: strong absorption (high alpha)



conduction band

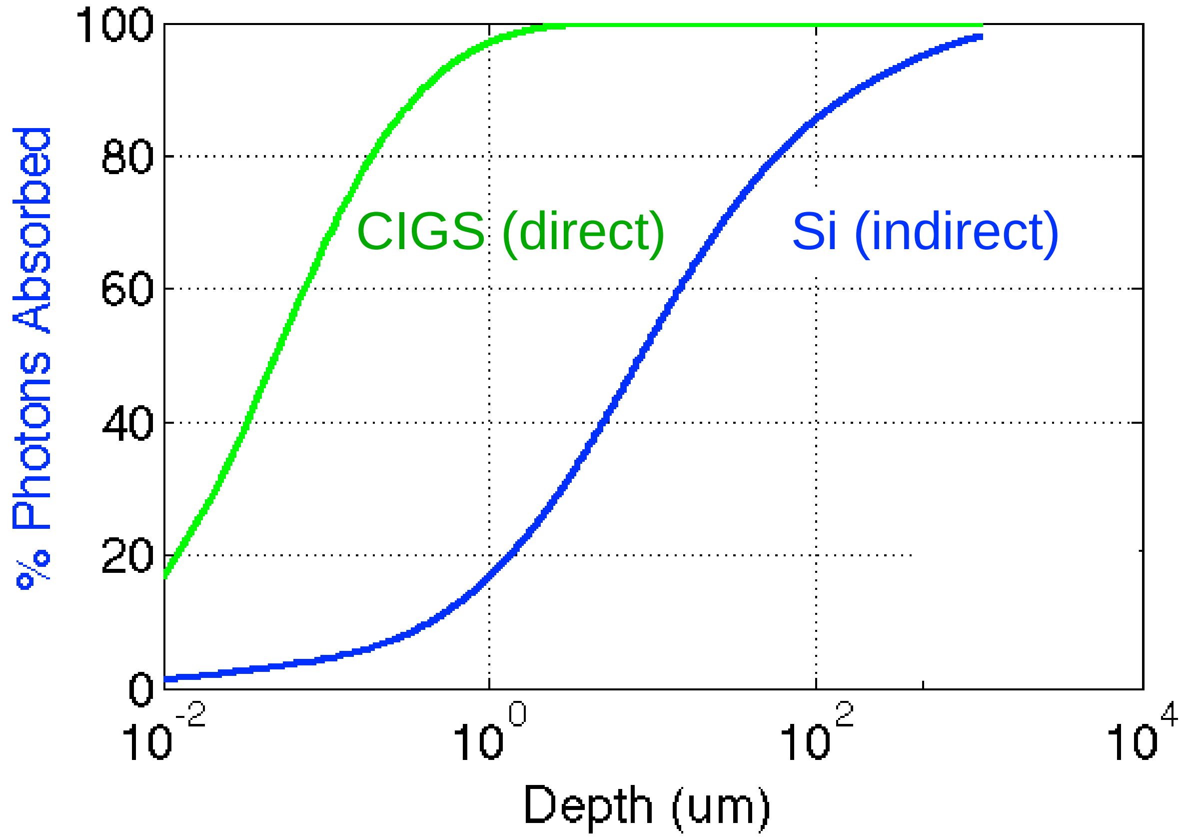
need phonon

valence band

Indirect gap: weaker absorption lower alpha)

light absorption vs. semiconductor thickness

The direct bandgap of CIGS allows it to absorb light much faster than Silicon. A layer of silicon must be 10^4 microns thick to absorb ~100% of the light, while CIGS need only be about 2 microns thick.



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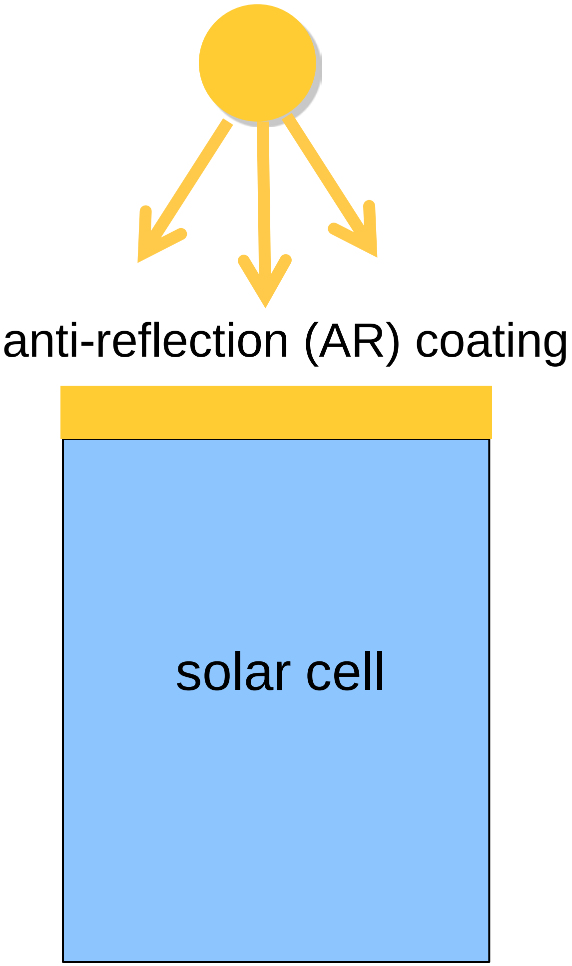
maximizing light absorption / generation

1) Maximize the number of phonons that get into the solar cell (AR coating, texturizing).

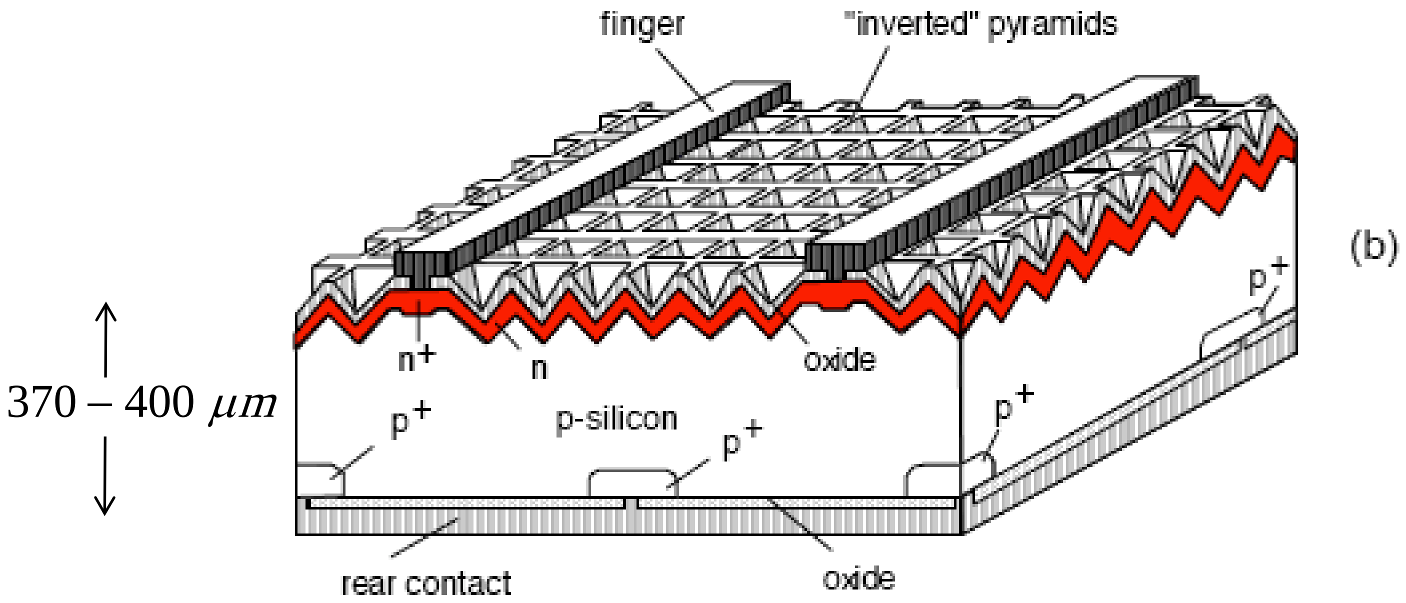
2) Maximize the "effective" thickness of the absorber.



anti-reflection (AR) coating

solar cell

light-trapping in high-efficiency Si solar cells

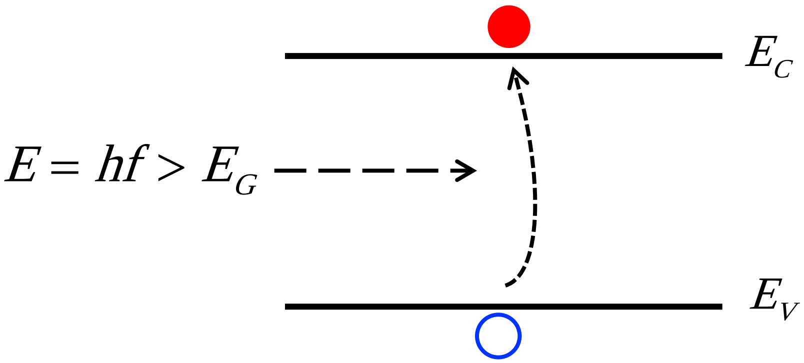


24.5% at 1 sun

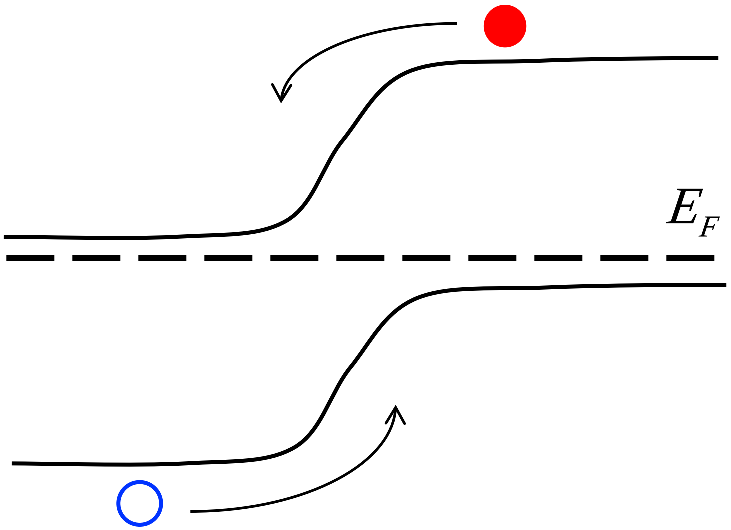
Martin Green Group UNSW – Zhao, et al, 1998

collection of e-h pairs

1) Light generates electron-hole pairs



2) PN junction collects e-h pairs

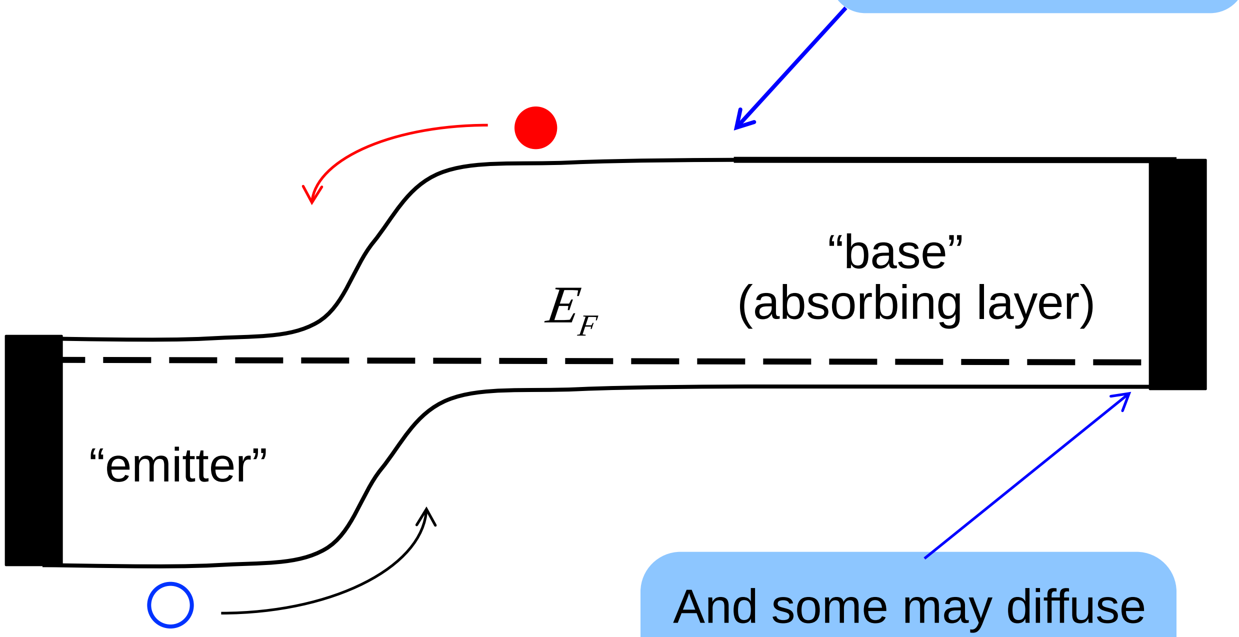


n-region collects the minority carrier electrons

p-region collects the minority carrier holes

collection efficiency

Photo-generated carriers should diffuse to the junction and be collected.



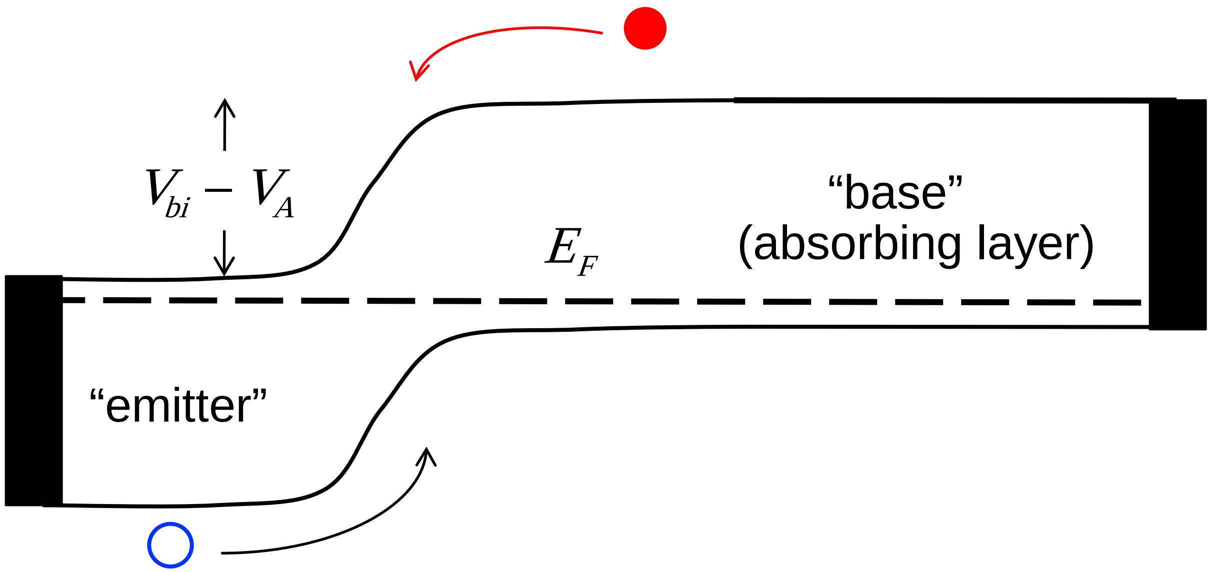
But some may recombine in the base.

And some may diffuse to the contact and recombine.

$$CE \equiv \frac{J_L}{J_{L\max}}$$

"collection efficiency"

voltage-dependence of collection

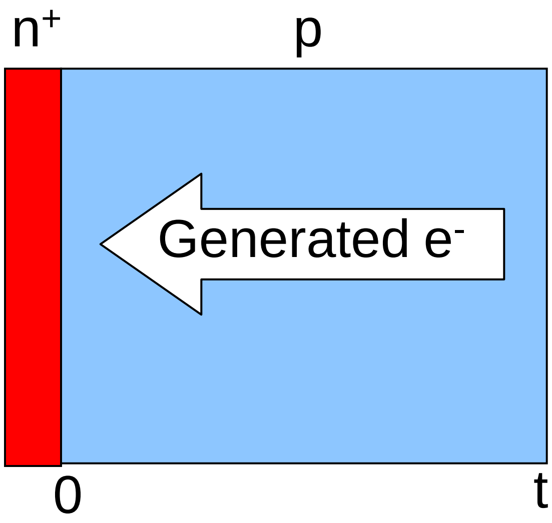


As long as $V_A < V_{bi}$, the applied bias should affect the collection of carriers.

current collection

The generated carriers are only useful to us if they are able to be collected.

Electron diffusion length: Distance e will travel before recombining



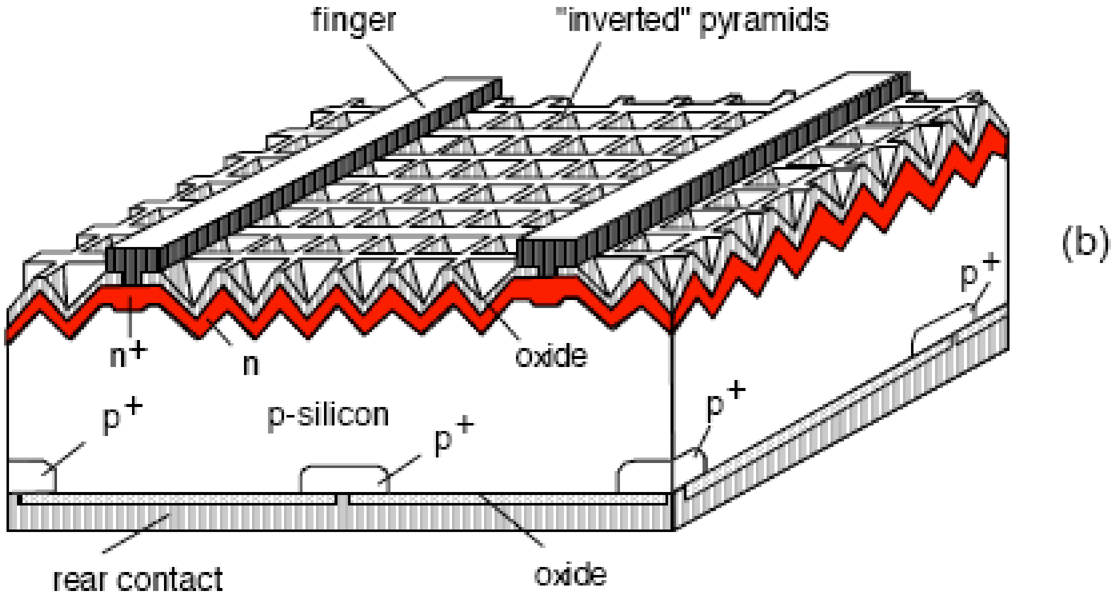
$$t < L_n = \sqrt{D_n \tau_n}$$

For Si: μ_n~1200 cm^2/V-s; τ_n~34 μs

L_n~320 μm

Therefore we cannot make the absorbing layer arbitrarily thick.

real thickness vs. optical thickness



24.5% at 1 sun

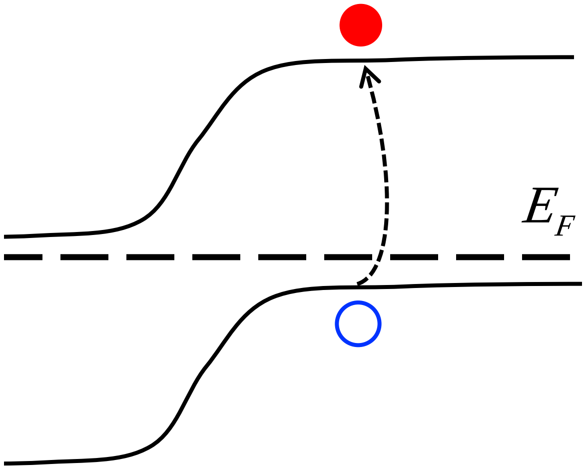
Martin Green Group UNSW – Zhao, et al, 1998

outline

1) Introduction
2) PN junction fundamentals (dark)
3) Model solar cell: dark IV
4) Optical absorption / light-generated current
5) **Model solar cell: illuminated**
6) Discussion
7) Summary

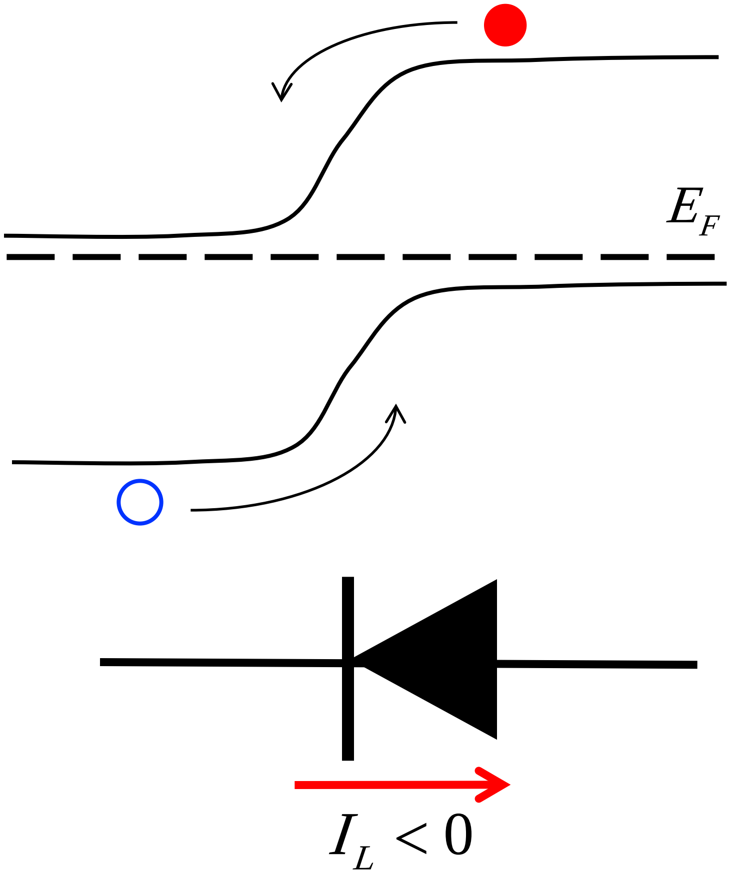
diode current under illumination

1) Light generates e-h pairs

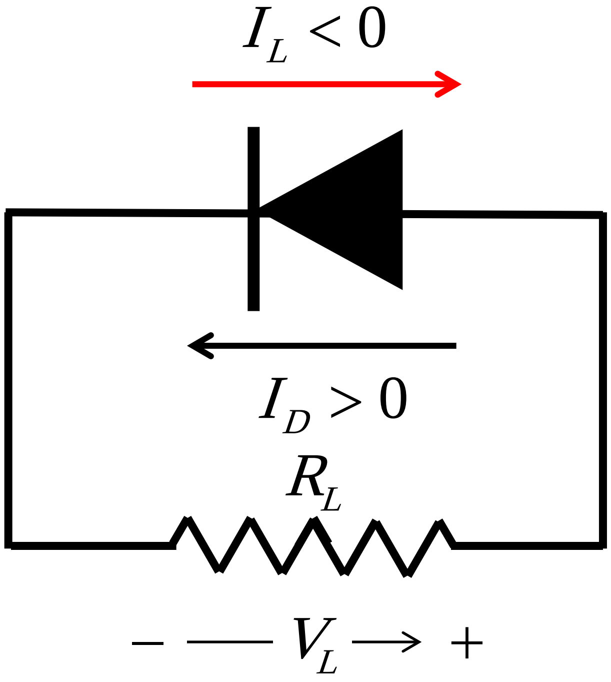


diode current under illumination

2) PN junction collects e-h pairs



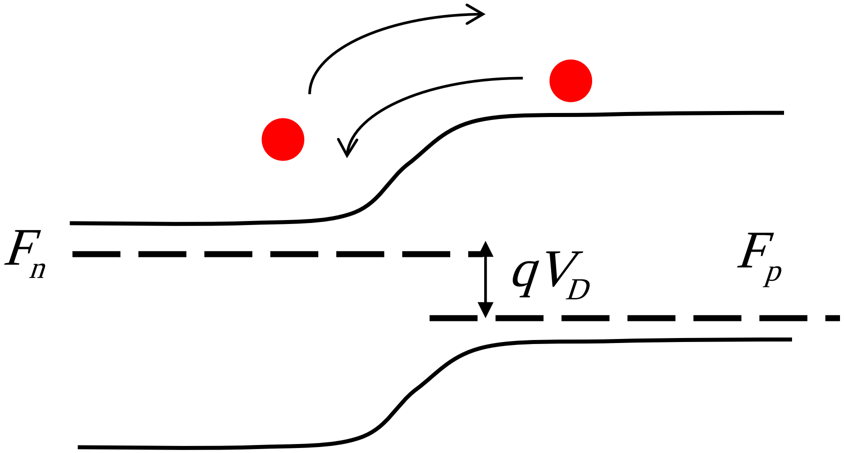
3) Current flows through load



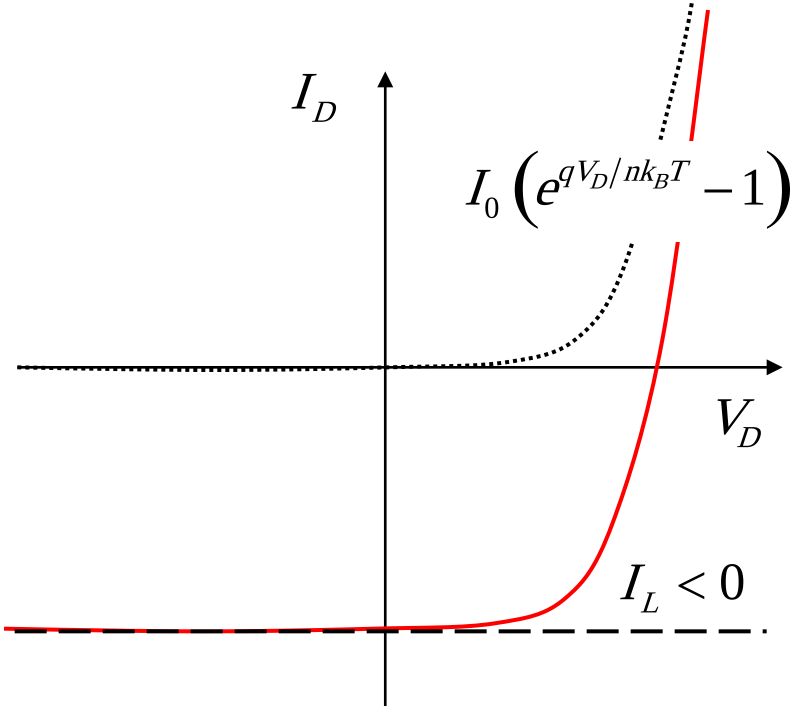
forward bias across PN junction develops

net current

4) Forward bias reduces current



5) IV characteristic is a superposition

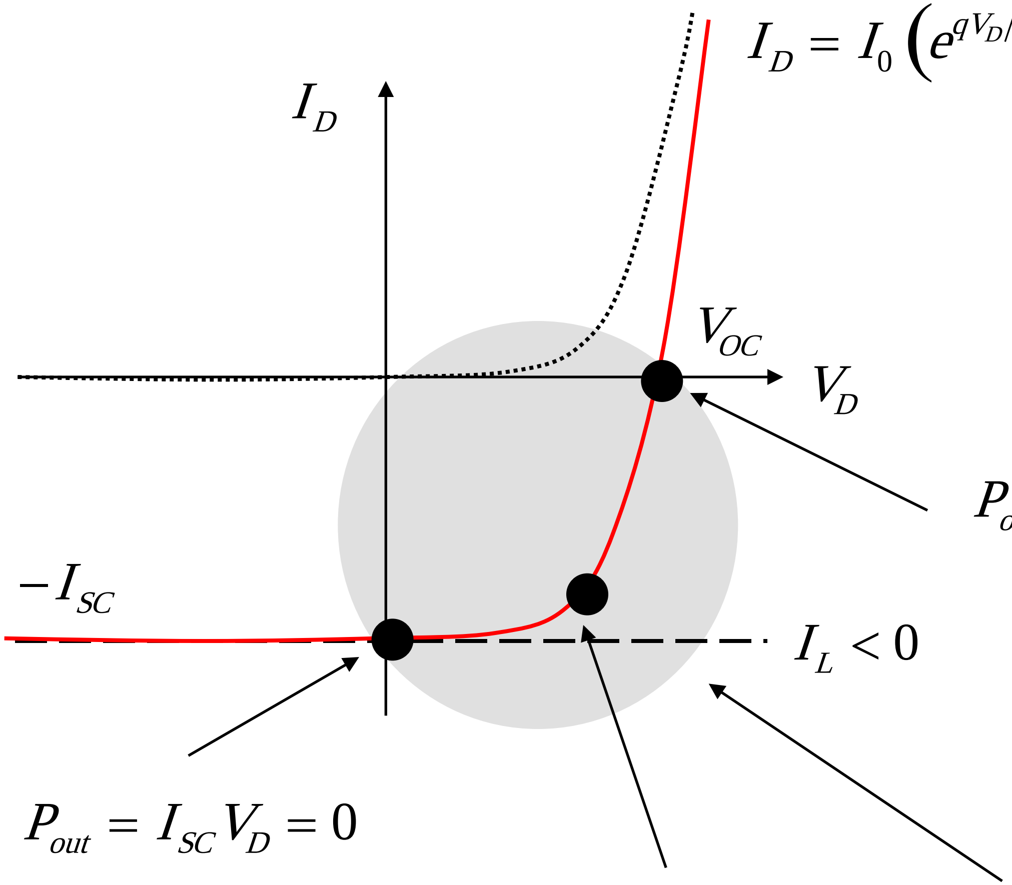


$$I_D = I_0\left(e^{qV_D/nk_BT}-1\right) - I_L$$

dark current - light-generated current

IV characteristic

6) Maximum power point



$$I_D = I_0\left(e^{qV_D/nk_BT} - 1\right) - I_L$$

$$\eta = \frac{P_{out}}{P_{in}} = \frac{I_{SC}V_{OC}FF}{P_{in}}$$

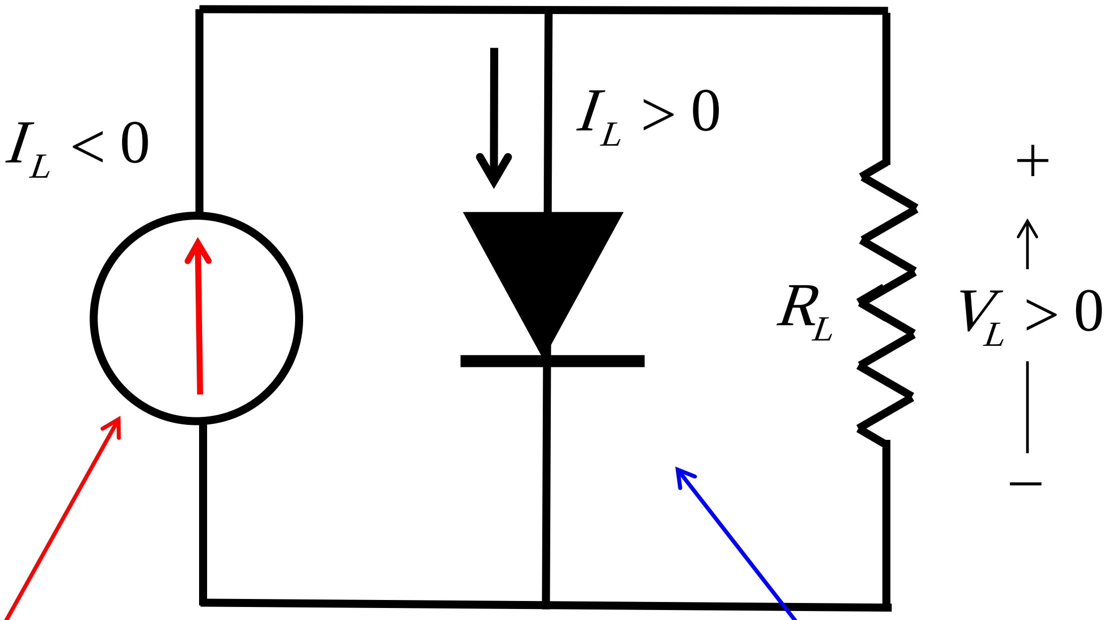
$$P_{out} = I_D V_{OC} = 0$$

$$P_{out} = I_{SC} V_D = 0$$

$$P_{out} = I_{mp} V_{mp} = I_{SC} V_{OC} FF$$

$$P_D = I_D V_D < 0$$

“superposition”

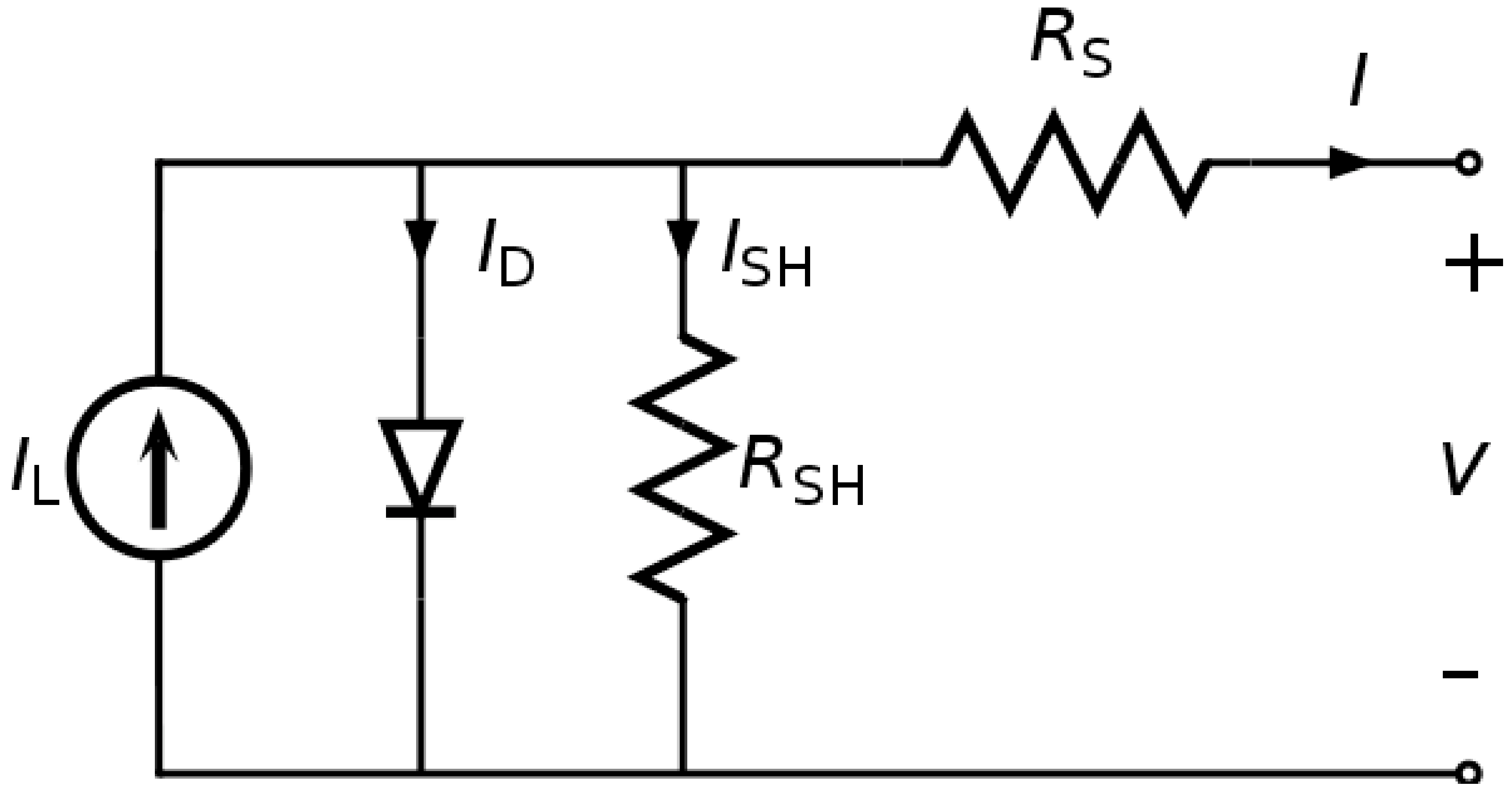


light-generated current
(bias independent)

dark current

$$I_D = I_0\left(e^{qV_D/nk_BT} - 1\right)$$

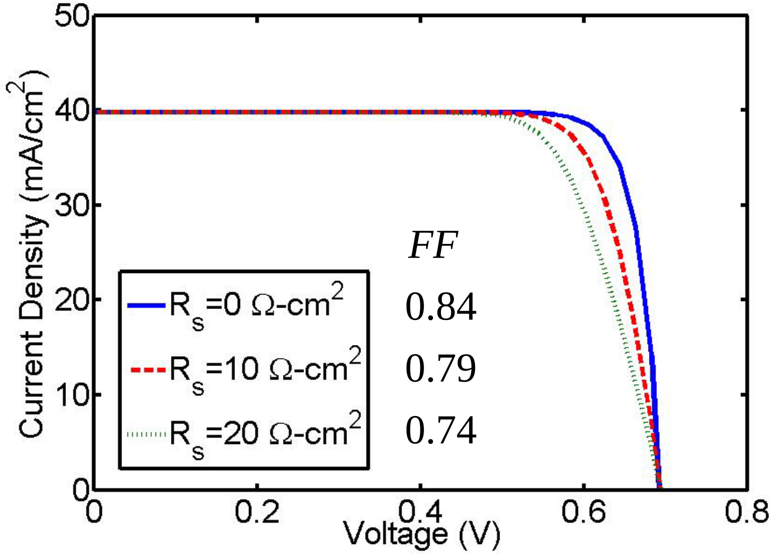
effect of series and shunt resistors



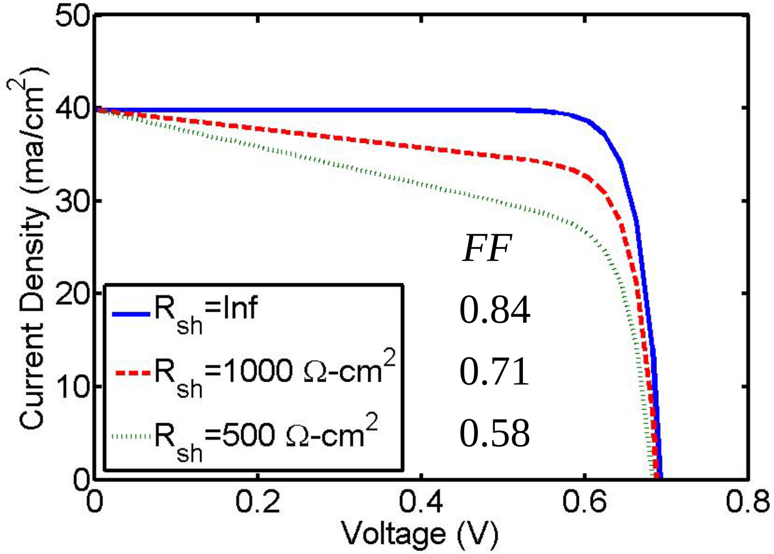
http://upload.wikimedia.org/wikipedia/commons/thumb/c/c4/Solar_cell_equivalent_circuit.svg/601px-Solar_cell_equivalent_circuit.svg.png

series and shunt resistors

Effect of series resistance



Effect of shunt resistance

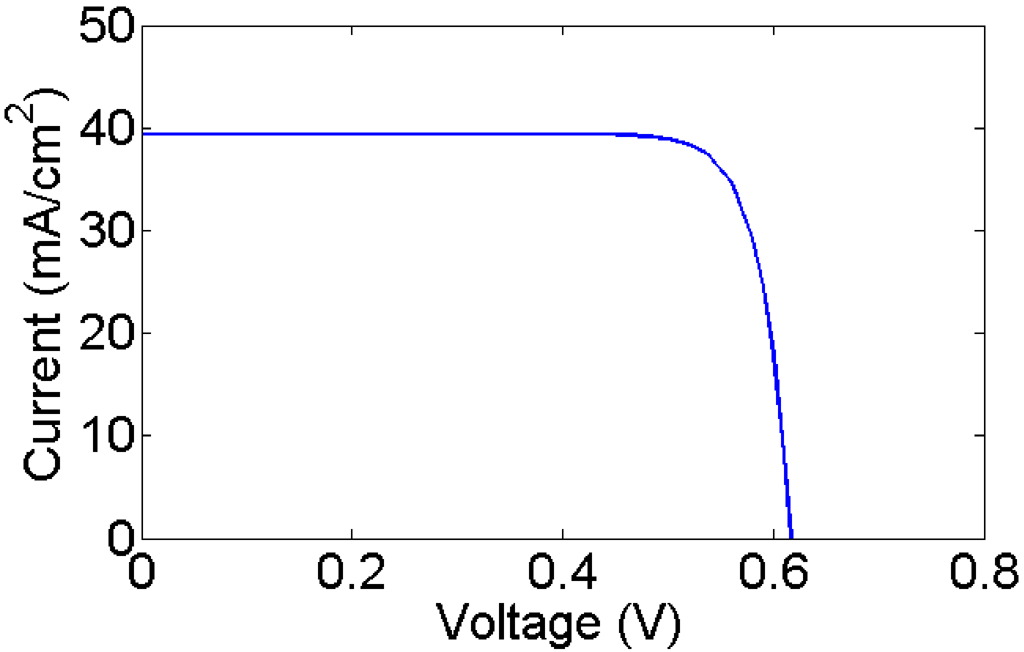


Modified Diode Equation:

$$I = I_L - I_0\left(e^{q\left(V + IR_S\right)/k_BT} - 1\right) - \frac{V + IR_S}{R_{SH}}$$

model solar cell simulation

ADEPT 2.0 nanoHUB.org



Results:

$$V_{OC} = 616 \text{ mV}$$

$$J_{SC} = 39.4 \text{ mA/cm}^2$$

$$FF = 0.83$$

$$\eta = 20.1\%$$

(See slide 26 for parameters)

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6) **Discussion**
    i) superposition
    ii) efficiency limits
    iii) costs
7) Summary

i) principle of superposition

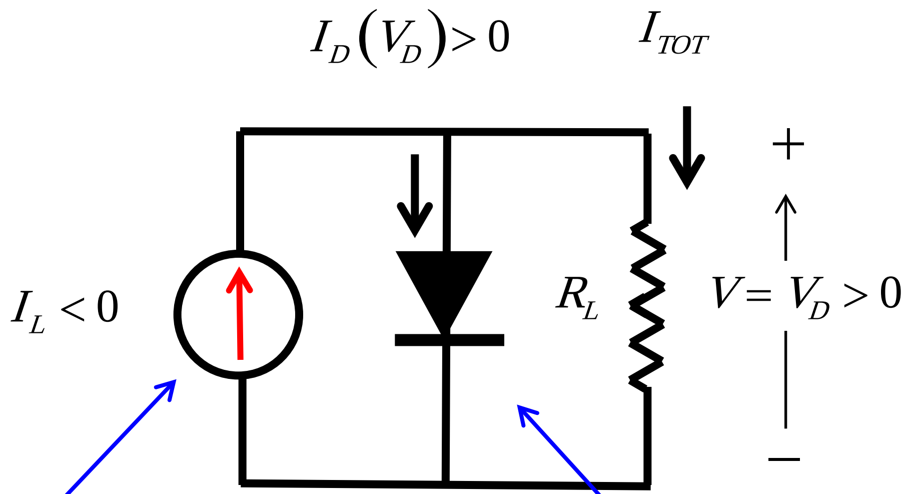


Photo-generated current collected by the junction. Generally assumed to be independent of the junction voltage.

$$I_{DARK} = I_0\left(e^{qV_D/nk_BT} - 1\right)$$

determined by recombination processes ***in the dark***

Superposition says that we can add the two solutions to get the total response of the solar cells.

superposition

We can superimpose two solutions when the differential equations describing the problem are ***linear***.

solar cell physics

“semiconductor equations”

Conservation Laws:

$$\nabla \bullet \vec{D} = \rho$$

$$\nabla \bullet \left(\vec{J}_n / -q \right) = \left(G - R \right)$$

$$\nabla \bullet \left(\vec{J}_p / \ q \right) = \left(G - R \right)$$

(steady-state)

Relations:

$$\vec{D} = \kappa \varepsilon_0 \vec{E} = -\kappa \varepsilon_0 \vec{\nabla} V$$

$$\rho = q\left(p - n + N_D^+ - N_A^- \right)$$

$$\vec{J}_n = nq\mu_n \vec{E} + qD_n \vec{\nabla} n$$

$$\vec{J}_p = pq\mu_p \vec{E} - qD_p \vec{\nabla} p$$

$$R = f\,(n, p)$$

$$G = \text{optical generation rate}$$

etc.

when does superposition apply?

1) the junction space-charge region may contribute importantly to either the photocurrent or the dark current, but not to both;
2) the carrier concentrations in the quasi-neutral regions must stay within low-injection levels;
3) the series resistance (and shunt resistance) must contribute negligilbly to the cell current-voltage characteristics;
4) the material parameters, such as the minority-carrier lifetime, must be essentially independent of the illumination level;
5) the volume of the regions that contribute appreciably to the photocurrent must stay essentially constant as the cell is loaded.

F. A. Lindholm, J.G. Fossum, and E.L. Burgess, “Application of the Superposition Principle to Solar-Cell Analysis,” *IEEE Trans. Electron Devices*, **26**, 165-171, 1979

ii) solar cell efficiency limits

$$\eta = \frac{P_{out}}{P_{in}} = \frac{I_{SC} V_{oc} FF}{P_{in}}$$

1) **Fill factor**:
   Determined by diode characteristic and series resistances.

2) **Short-circuit current**:
   Increases as the bandgap decreases.

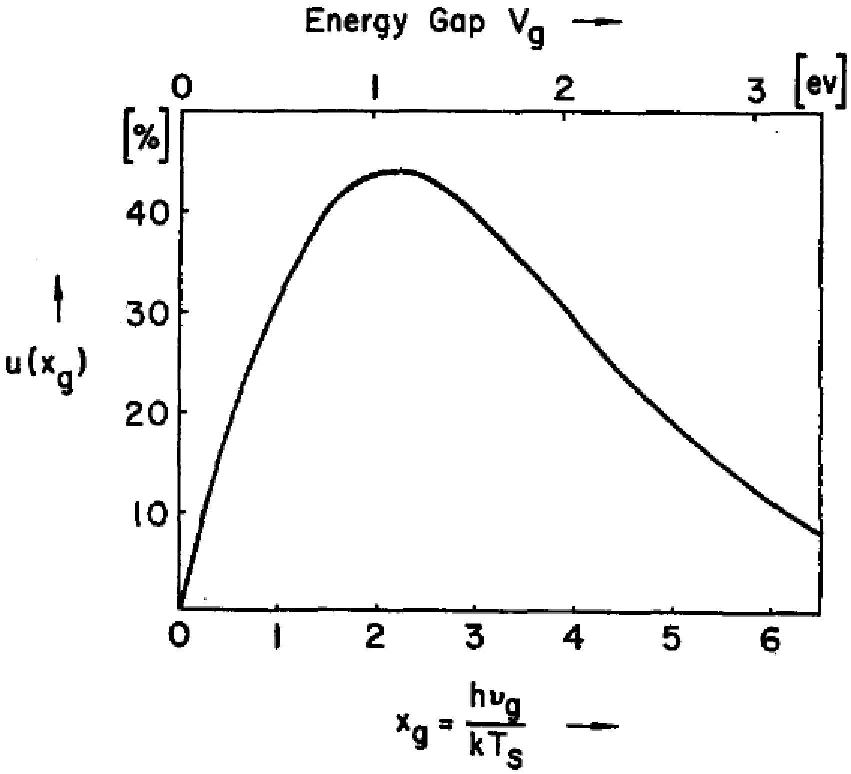
   For a given bandgap, determined by reflection, absorption, recombination.

3) **Open-circuit voltage**:
   Increases as the bandgap increases.

   For a given bandgap, determined by recombination.

Shockley-Queisser Limit

1) Smaller bandgaps give higher short circuit current
2) Larger bandgaps give higher open-circuit voltage
3) For the given solar spectrum, an optimum bandgap exists.



W. Shockley, and H. J. Queisser, "Detailed Balance Limit of Efficiency of p-n Junction Solar Cells", *J. Appl. Phys.,* **32**, 510, 1961.

iii) costs: three approaches

1) High-efficiency , but high-cost solar cells
   (high-quality crystalline materials)

2) Acceptable efficiency, but very low costs
   (thin-film, amorphous/polycrystalline materials)

3) Concentration
   (high efficiency cells with optical concentration)

“grid parity”

For a photovoltaic power generation system to be economically competitive the total costs of an installed PV system must be ~ $1/W, which translates to 5-6 cents per kilowatt-hour.

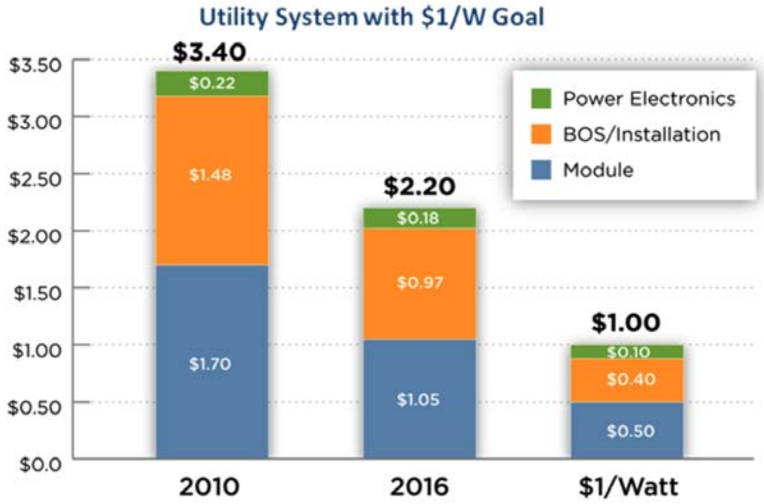
A system includes:

1) Module costs
2) Power conditioning electronics
3) Installation and balance of systems.

Current costs are $3.40/W (2011).

Projected to decrease to $2.20/W by 2016

the dollar per Watt goal



The U.S. Department of Energy is calling for research to meet the $1/W goal. This requires modules at $0.50/W, with ~20% modules with 20-30 system lifetimes.

http://www1.eere.energy.gov/solar/

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solar cell summary

1) Light is absorbed and produces e-h pairs

2) PN junctions separate e-h pairs and collect the carriers.

3) Current flow in external circuit produces a FB voltage and the FB diode current reduces the total current.

4) Power out is $I_{SC} V_{OC} FF$.

5) Unlike integrated circuit chips, where the value added comes from the design/system, manufacturing costs are critical in PV.

references

Textbooks:

Martin Green, *Solar Cells: Operating Principles, Technology, and System Applications*, Prentice-Hall, 1981.

Jenny Nelson, *Physics of Solar Cells*, Imperial Collage Press, 2003

Antonio Lugue and Steven Hegedus, *Handbook of Photovoltaic Science and Engineering*, John Wiley and Sons, 2003.

Compilation of current cell efficiencies:

Martin Green, *et al.*, "Solar cell efficiency tables (version 37)", **19**, *Prog. in Photovoltaics*, 2011

Survey of 3rd generation PV concepts:

Martin Green, *Third generation photovoltaics: advanced solar energy conversion*, Springer, 2006.

questions

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